

Warm-up #8: Solving Trig Equations

Ex. Solve: $[0, 2\pi)$

$$\begin{aligned}
 1. \quad 3 &= 3\csc\theta - \cot^2\theta \\
 3 &= 3\csc\theta - (\csc^2\theta - 1) \\
 3 &= 3\csc\theta - \csc^2\theta + 1 \\
 \cancel{3} & \qquad \qquad \qquad \cancel{-3} \\
 \hline
 0 &= 3\csc\theta - \csc^2\theta - 2 \\
 \csc^2\theta - 3\csc\theta + 2 &= 0 \\
 (\csc\theta - 2)(\csc\theta - 1) &= 0 \\
 \csc\theta - 2 = 0 \quad \csc\theta &= 1 \\
 \csc\theta = 2 \quad \sin\theta &= \frac{1}{2} \\
 \sin\theta = \frac{1}{2} &
 \end{aligned}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{2}$$

$$\begin{aligned}
 2. \quad (\sqrt{3}\sin\theta)^2 &= (\cos\theta)^2 \\
 3\sin^2\theta &= \cos^2\theta \\
 3(1 - \cos^2\theta) &= \cos^2\theta \\
 3 - 3\cos^2\theta &= \cos^2\theta \\
 +3\cos^2\theta \quad +3\cos^2\theta &
 \end{aligned}$$

$$3 = 4\cos^2\theta$$

$$\sqrt{\frac{3}{4}} = \sqrt{\cos^2\theta}$$

$$\pm \frac{\sqrt{3}}{2} = \cos\theta$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$