Warmup 4: Double and Half Angle Identities

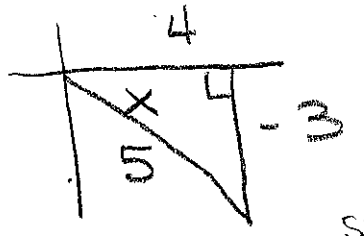
Given $\cos x = \frac{4}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, find:

a) $\sin 2x$

b) $\sin\frac{x}{2}$

c) $\cos\frac{x}{2}$

d) $\tan 2x$



S | A
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$\frac{\frac{3\pi}{2}}{2} < \frac{x}{2} < \frac{2\pi}{2} \rightarrow \frac{3\pi}{4} < \frac{x}{2} < \pi$

a) $\frac{-24}{25}$

b) $\frac{\sqrt{10}}{10}$

c) $\frac{-3\sqrt{10}}{10}$

d) $\frac{-24}{7}$

Find the exact value using the double or half angle identities:

e) $\cos\left(\frac{7\pi}{3}\right)$

f) $\tan(105°)$

e) $\frac{1}{2}$

f) $-2-\sqrt{3}$

a.) $\sin 2x = 2\sin x\cos x = 2\cdot\frac{-3}{5}\cdot\frac{4}{5} = \boxed{\frac{-24}{25}}$

b.) $\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-4/5}{2}} = \sqrt{\frac{1/5}{2}} = \sqrt{\frac{1}{5}\cdot\frac{1}{2}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$

c.) $\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+4/5}{2}} = -\sqrt{\frac{9/5}{2}} = -\sqrt{\frac{9}{5}\cdot\frac{1}{2}} = -\sqrt{\frac{9}{10}} = -\frac{3}{\sqrt{10}} = \boxed{\frac{-3\sqrt{10}}{10}}$

d.) $\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot -3/4}{1-(-3/4)^2} = \frac{-6/4}{\frac{16}{16}-\frac{9}{16}} = \frac{-3/2}{7/16} = -\frac{3}{2}\cdot\frac{16^8}{7} = \boxed{\frac{-24}{7}}$

e.) $\cos\frac{7\pi}{3} = \cos\left(2\cdot\frac{7\pi}{6}\right)$

$1-2\sin^2 x$

$1-2\sin^2\left(\frac{7\pi}{6}\right)$

$1-2\left(-\frac{1}{2}\right)^2$

$1-2\cdot\frac{1}{4}$

$1-\frac{1}{2}$

$\boxed{\frac{1}{2}}$

f.) $\tan 105° = \tan\left(\frac{210°}{2}\right)$

$= \frac{1-\cos x}{\sin x}$

$= \frac{1-\cos 210°}{\sin 210°}$

$= \frac{1-(-\sqrt{3}/2)}{-1/2} = \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{-1/2} = \frac{\frac{2+\sqrt{3}}{2}}{-1/2}$

$= \frac{2+\sqrt{3}}{2}\cdot\frac{-2}{1} = \boxed{-2-\sqrt{3}}$ or $\boxed{-\sqrt{3}-2}$