1. $\sin^3\theta + \sin\theta\cos^2\theta = \sin\theta$

$\sin\theta(\sin^2\theta + \cos^2\theta)$

$\sin\theta(1)$

$\sin\theta$ ✓

2. $\dfrac{1+\sec(-x)}{\sin(-x)+\tan(-x)} = -\csc x$

$\dfrac{1+\sec x}{-\sin x - \tan x}$

$\dfrac{1+\sec x}{-(\sin x + \tan x)}$

$\dfrac{-(1+\sec x)}{\sin x + \frac{\sin x}{\cos x}}$

$\dfrac{-(1+\sec x)}{\sin x\left(1+\frac{1}{\cos x}\right)}$

$\dfrac{-(1+\sec x)}{\sin x(1+\sec x)}$

$\dfrac{-1}{\sin x}$

$-\csc x$ ✓

3. $\sec x + \tan x = \dfrac{\cos x}{1-\sin x}$

mult by $(1+\sin x)$

$= \dfrac{\cos x(1+\sin x)}{1-\sin^2 x}$

$= \dfrac{\cos x(1+\sin x)}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos x}$

$= \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$

$= \sec x + \tan x$ ✓

4. $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

mult by $\cos x + \cos y$, $\sin x + \sin y$

$$\frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{\cos^2 x + \sin^2 x - \cos^2 y - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{1 - 1}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{0}{(\sin x + \sin y)(\cos x + \cos y)}$$

$0$ ✓

5. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x$ ✓

6. $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

$$\frac{\cot^2 x}{\csc^2 x}$$

$$\frac{\cos^2 x / \sin^2 x}{1/\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1}$$

$\cos^2 x$ ✓

7. $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$

mult. by $1+\cos x$, $1-\cos x$

$$\frac{1+\cos x + 1-\cos x}{(1-\cos x)(1+\cos x)}$$

$$\frac{2}{1-\cos^2 x}$$

$$\frac{2}{\sin^2 x}$$

$2\csc^2 x$ ✓

8. $(\cot^2\theta+1)(\sin^2\theta-1) = -\cot^2\theta$

$\csc^2\theta \cdot -\cos^2\theta$

$\frac{1}{\sin^2\theta} \cdot -\cos^2\theta$

$$\frac{-\cos^2\theta}{\sin^2\theta}$$

$-\cot^2\theta$ ✓

9. $\csc x + \cot x = \frac{\sin x}{1-\cos x}$

mult. by $1+\cos x$

$$\frac{\sin x(1+\cos x)}{1-\cos^2 x}$$

$$\frac{\sin x(1+\cos x)}{\sin^2 x}$$

$$\frac{1+\cos x}{\sin x}$$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$\csc x + \cot x$ ✓

10. $\frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y} = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

$$\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x}\frac{\sin y}{\cos y}}$$

mult by $\cos y$ ↗ ↖ mult by $\cos x$

mult by $\cos x \cos y$ ↗

$$\frac{\frac{\sin x\cos y + \sin y\cos x}{\cos x\cos y}}{\frac{\cos x\cos y - \sin x\sin y}{\cos x\cos y}}$$

$$\frac{\sin x\cos y + \sin y\cos x}{\cos x\cos y} \cdot \frac{\cos x\cos y}{\cos x\cos y - \sin x\sin y}$$

$$\frac{\sin x\cos y + \sin y\cos x}{\cos x\cos y - \sin x\sin y}$$

✓