

$$1. \cos^2 A \cdot \csc A \cdot \sec A = \cot A$$

$$\cos^2 A \cdot \frac{1}{\sin A} \cdot \frac{1}{\cos A}$$

$$\frac{\cos^2 A}{\sin A \cos A}$$

$$\frac{\cos A}{\sin A}$$

$$\cot A \checkmark$$

$$2. \tan B (\sin B + \cot B \cdot \cos B) = \sec B$$

$$\frac{\sin B}{\cos B} (\sin B + \frac{\cos B}{\sin B} \cdot \cos B)$$

$$\frac{\sin^2 B}{\cos B} + \frac{\sin B \cos^2 B}{\sin B \cos B}$$

$$\frac{\sin^2 B}{\cos B} + \frac{\cos B}{1}$$

← mult. by cos

$$\frac{\sin^2 B + \cos^2 B}{\cos B}$$

$$\frac{\sin^2 B + \cos^2 B}{\cos B}$$

$$\frac{1}{\cos B}$$

$$\sec B \checkmark$$

$$3. \cos x (\sec x + \cos x \csc^2 x) = \csc^2 x$$

$$\cos x \left(\frac{1}{\cos x} + \frac{\cos x}{\sin^2 x} \right)$$

$$\frac{\cos x}{\cos x} + \frac{\cos^2 x}{\sin^2 x}$$

$$1 + \cot^2 x$$

$$\csc^2 x \checkmark$$

$$4. (\cos x - \sin x)^2 = 1 - 2 \sin x \cos x$$

$$(\cos x - \sin x)(\cos x - \sin x)$$

$$\cos^2 x - 2 \sin x \cos x + \sin^2 x$$

$$1 - 2 \sin x \cos x \checkmark$$

$$5. (\tan B + \cot B)^2 = \sec^2 B + \csc^2 B$$

$$(\tan B + \cot B)(\tan B + \cot B)$$

$$\tan^2 B + 2 \tan B \cot B + \cot^2 B$$

$$\tan^2 B + 2 + \cot^2 B$$

$$\sec^2 B + 2 + \csc^2 B$$

$$\sec^2 B + \csc^2 B \checkmark$$

$$6. \frac{1 + \cot^2 x}{\sec^2 x} = \cot^2 x$$

$$\frac{\csc^2 x}{\sec^2 x}$$

$$\frac{1/\sin^2 x}{1/\cos^2 x}$$

$$\frac{1}{\sin^2 x} \cdot \frac{\cos^2 x}{1}$$

$$\frac{\cos^2 x}{\sin^2 x}$$

$$\cot^2 x \checkmark$$

$$7 \quad \frac{\sec A}{\sin A} - \frac{\sin A}{\cos A} = \cot A$$

↑ mult. by $\cos A$ ↑ mult. by $\sin A$

$$\frac{\sec A \cos A}{\sin A \cos A} - \frac{\sin A \sin A}{\sin A \cos A}$$

$$\frac{1}{\sin A \cos A} - \frac{\sin^2 A}{\sin A \cos A}$$

$$\frac{1 - \sin^2 A}{\sin A \cos A}$$

$$\frac{\cos^2 A}{\sin A \cos A}$$

$$\frac{\cos A}{\sin A}$$

$$\cot A \checkmark$$

$$8. \quad \frac{1}{1 - \cos y} + \frac{1}{1 + \cos y} = 2 \csc^2 y$$

$$\frac{1}{1 - \cos y} \quad + \quad \frac{1}{1 + \cos y}$$

↓ mult. by $1 + \cos y$ ↑ mult. by $1 - \cos y$

$$\frac{1 + \cos y}{(1 - \cos y)(1 + \cos y)} + \frac{1 - \cos y}{(1 - \cos y)(1 + \cos y)}$$

$$\frac{2}{(1 - \cos y)(1 + \cos y)}$$

$$\frac{2}{1 - \cos^2 y}$$

$$\frac{2}{\sin^2 y}$$

$$2 \csc^2 y \checkmark$$

$$9. \cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$$

$$\cot^2 x (\csc^2 x - 1)$$

$$\frac{\cot^2 x \cdot \cot^2 x}{\cot^4 x} \checkmark$$

$$10. \sec^4 a - \tan^4 a = 1 + 2 \tan^2 a$$

$$(\sec^2 a + \tan^2 a)(\sec^2 a - \tan^2 a)$$

$$(\sec^2 a + \tan^2 a)(1)$$

$$\sec^2 a + \tan^2 a$$

$$\tan^2 a + 1 + \tan^2 a$$

$$1 + 2 \tan^2 a \checkmark$$

$$11. \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} = \tan x$$

↑ mult. by cos

$$\frac{1}{\sin x \cos x} - \frac{\cos^2 x}{\sin x \cos x}$$

$$\frac{1 - \cos^2 x}{\sin x \cos x}$$

$$\frac{\sin^2 x}{\sin x \cos x}$$

$$\frac{\sin x}{\cos x}$$

$$\tan x \checkmark$$

$$12. \frac{1}{1 - \sin r} = \sec^2 r + \sec r \tan r$$

↑ multiply by conjugate (1 + sin r)

$$\frac{(1 + \sin r)}{(1 - \sin r)(1 + \sin r)}$$

$$\frac{1 + \sin r}{1 - \sin^2 r}$$

$$\frac{1 + \sin r}{\cos^2 r}$$

$$\frac{1}{\cos^2 r} + \frac{\sin r}{\cos^2 r}$$

Sorry!

$$\sec^2 r + \left(\frac{\sin r}{\cos r} \cdot \frac{1}{\cos r} \right)$$

$$\sec^2 r + \tan r \cdot \sec r$$

$$\sec^2 r + \sec r \tan r \checkmark$$

$$13 \quad \frac{\cos x}{\sec x - 1} - \frac{\cos x}{\tan^2 x} = \cot^2 x$$

↑
mult. by $\sec x + 1$

$$\frac{\cos x (\sec x + 1)}{\sec^2 x - 1} - \frac{\cos x}{\tan^2 x}$$

$$\frac{1 + \cos x}{\tan^2 x} - \frac{\cos x}{\tan^2 x}$$

$$\frac{1}{\tan^2 x}$$

$$\cot^2 x \checkmark$$

$$14. \quad \frac{\sec x}{\sec x - \tan x} = \sec^2 x + \sec x \tan x$$

$$\frac{\sec x - \tan x}{\sec x - \tan x}$$

↑
mult. by $\sec x + \tan x$

$$\frac{\sec x (\sec x + \tan x)}{\sec^2 x - \tan^2 x}$$

$$\frac{\sec^2 x + \sec x \tan x}{1}$$

$$\sec^2 x + \sec x \tan x \checkmark$$

$$15. \quad \frac{1 + \sin x}{1 - \sin x} = 2 \sec^2 x + 2 \sec x \tan x - 1$$

↑
mult. by $1 + \sin x$

$$\frac{(1 + \sin x)^2}{1 - \sin^2 x}$$

$$\frac{1 + 2 \sin x + \sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x} + \frac{2 \sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$\sec^2 x + \frac{2 \sin x}{\cos x} \cdot \frac{1}{\cos x} + \tan^2 x$$

$$\sec^2 x + 2 \tan x \cdot \sec + \tan^2 x$$

$$\sec^2 x + 2 \sec x \tan x + \sec^2 x - 1$$

$$2 \sec^2 x + 2 \sec x \tan x - 1 \checkmark$$

$$\begin{aligned}
 16. \sin^3 y \cos^2 y &= \sin^3 y - \sin^5 y \\
 &= \sin^3 y (1 - \sin^2 y) \\
 &= \sin^3 y \cos^2 y \checkmark
 \end{aligned}$$

$$17. \sec^2 \theta + \csc^2 \theta = \sec^2 \theta \csc^2 \theta$$

$$\begin{aligned}
 &\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \\
 &\quad \uparrow \text{mult by } \sin^2 \theta, \quad \uparrow \text{mult by } \cos^2 \theta \\
 &\frac{\sin^2 \theta}{\cos^2 \theta \sin^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta \sin^2 \theta} \\
 &\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta} \\
 &\frac{1}{\cos^2 \theta \sin^2 \theta} \\
 &\sec^2 \theta \csc^2 \theta \checkmark
 \end{aligned}$$

$$\begin{aligned}
 18. \sec \theta + \tan \theta &= \frac{1}{\sec \theta - \tan \theta} \\
 &\quad \uparrow \text{mult by } \sec \theta + \tan \theta \\
 &= \frac{\sec \theta + \tan \theta}{\sec^2 \theta - \tan^2 \theta} \\
 &= \frac{\sec \theta + \tan \theta}{1} \\
 &= \sec \theta + \tan \theta \checkmark
 \end{aligned}$$

$$\begin{aligned}
 19. \frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} &= \frac{1 - 4\cos x}{1 - \cos x} \\
 &\quad \uparrow \text{mult by } 1 + \cos x \\
 &= \frac{(1 - 4\cos x)(1 + \cos x)}{1 - \cos^2 x} \\
 &= \frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 20. \frac{\sec^2 x - 6 \tan x + 7}{\sec^2 x - 5} &= \frac{\tan x - 4}{\tan x + 2} \\
 &\quad \uparrow \text{mult. by } \tan x - 2 \\
 &= \frac{-(\tan x - 4)(\tan x - 2)}{\tan^2 x - 4} \\
 &= \frac{\tan^2 x - 6 \tan x + 8}{\tan^2 x - 4} \\
 &= \frac{\sec^2 x - 1 - 6 \tan x + 8}{\sec^2 x - 1 - 4} \\
 &= \frac{\sec^2 x - 6 \tan x + 7}{\sec^2 x - 5} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 21. \frac{\sec^3 x - \cos^3 x}{\sec x - \cos x} &= \sec^2 x + 1 + \cos^2 x \\
 &\quad \uparrow \text{mult. by } \sec x + \cos x \\
 &= \frac{(\sec^3 x - \cos^3 x)(\sec x + \cos x)}{\sec^2 x - \cos^2 x} \\
 &= \frac{\sec^4 x + \cos x \sec^3 x - \cos^3 x \sec x - \cos^4 x}{\sec^2 x - \cos^2 x} \\
 &= \frac{(\sec^4 x - \cos^4 x) + (\cos x \sec^3 x - \cos^3 x \sec x)}{\sec^2 x - \cos^2 x} \\
 &= \frac{(\sec^2 x - \cos^2 x)(\sec^2 x + \cos^2 x) + \cos x \sec x (\sec^2 x - \cos^2 x)}{\sec^2 x - \cos^2 x} \\
 &= \frac{(\sec^2 x - \cos^2 x)(\sec^2 x + \cos^2 x + \cos x \sec x)}{\sec^2 x - \cos^2 x} \\
 &= \sec^2 x + \cos^2 x + 1 \\
 &= \sec^2 x + 1 + \cos^2 x \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & (2 \sin x + 3 \cos x)^2 + (3 \sin x - 2 \cos x)^2 = 13 \\
 & 4 \sin^2 x + 12 \sin x \cos x + 9 \cos^2 x + 9 \sin^2 x - 12 \sin x \cos x + 4 \cos^2 x \\
 & 4 \sin^2 x + 4 \cos^2 x + 9 \cos^2 x + 9 \sin^2 x \\
 & 4(\sin^2 x + \cos^2 x) + 9(\cos^2 x + \sin^2 x) \\
 & 4(1) + 9(1) \\
 & 4 + 9 \\
 & 13 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{1 + \sin x + \cos x}{1 + \sin x - \cos x} - \frac{1 + \cos x}{\sin x} \\
 & \frac{(1 + \sin x + \cos x)(1 - \sin x + \cos x)}{(1 + \sin x - \cos x)(1 - \sin x + \cos x)} \\
 & \frac{1 - \sin^2 x + \cos^2 x + \sin x - \sin^2 x + \cos x \sin x + \cos x - \cos x \sin x + \cos^2 x}{1 - \sin^2 x + \cos^2 x + \sin x - \sin^2 x + \cos x \sin x - \cos x \sin x + \cos^2 x} \\
 & \frac{1 + 2 \cos^2 x - \sin^2 x + \cos^2 x}{1 - \sin^2 x - \cos^2 x + 2 \cos x \sin x} \\
 & \frac{1 + 2 \cos^2 x - (1 - \cos^2 x) + \cos^2 x}{1 - (\sin^2 x + \cos^2 x) + 2 \cos x \sin x} \\
 & \frac{1 + 2 \cos^2 x - 1 + \cos^2 x + \cos^2 x}{1 - 1 + 2 \cos x \sin x} \\
 & \frac{2 \cos^2 x + 2 \cos^2 x}{2 \cos x \sin x} \\
 & \frac{2 \cos^2 x (1 + \cos x)}{2 \cos x \sin x} \\
 & \frac{1 + \cos x}{\sin x} \checkmark
 \end{aligned}$$

$$24. \frac{1 + \sin x + \cos x}{1 - \sin x + \cos x} = \frac{1 + \sin x}{\cos x}$$

$$\frac{(1 + \sin x + \cos x)(1 + \sin x - \cos x)}{(1 - \sin x + \cos x)(1 + \sin x - \cos x)}$$

$$\frac{1 + \sin x - \cos x + \sin x + \sin^2 x - \cos x \sin x + \cos x + \cos x \sin x - \cos^2 x}{1 + \sin x - \cos x - \sin x - \sin^2 x + \cos x \sin x + \cos x + \cos x \sin x - \cos^2 x}$$

$$\frac{1 + 2\sin x + \sin^2 x - \cos^2 x}{1 + 2\sin x + \sin^2 x - (1 - \sin^2 x)}$$

$$\frac{1 - \sin^2 x - \cos^2 x + 2\cos x \sin x}{1 + 2\sin x + \sin^2 x - (1 - \sin^2 x)}$$

$$\frac{2\cos x \sin x}{1 + 2\sin x + \sin^2 x - 1 + \sin^2 x}$$

$$\frac{2\cos x \sin x}{2\sin^2 x + 2\sin x}$$

$$\frac{2\cos x \sin x}{2\sin x (\sin x + 1)}$$

$$\frac{2\cos x \sin x}{2\cos x \sin x}$$

$$\frac{1 + \sin x}{\cos x} \quad \checkmark$$