Verifying Trig Identities
with Double & Half Angles

Name Key

First, some extra practice with writing each expression as a single trig function.

Write each expression in terms of a single trigonometric function.
(BTW … All angles below are radians, but that doesn't change your thought process!)

1. $2\sin 0.6\cos 0.6$
   $\sin(2\cdot.6)=\sin(1.2)$

2. $2\sin 3\cos 3$
   $\sin(2\cdot 3)=\sin(6)$

3. $2\sin 2\cos 2$
   $\sin(2\cdot 2)=\sin(4)$

4. $\cos^2 0.45-\sin^2 0.45$
   $\cos(2\cdot.45)=\cos(.9)$

5. $2\cos^2 5-1$
   $\cos(2\cdot 5)=\cos(10)$

6. $1-2\sin^2 3$
   $\cos(2\cdot 3)=\cos(6)$

7. $2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$ $=\sin(2\cdot\frac{\pi}{6})=\sin(\frac{\pi}{3})$

8. $\cos^2\frac{\pi}{10}-\sin^2\frac{\pi}{10}$ $=\cos(2\cdot\frac{\pi}{10})=\cos(\frac{\pi}{5})$

Verify each identity:

9. $1+\sin 2\theta=(\sin\theta+\cos\theta)^2$

10. $\sin 2\theta=2\cot\theta\sin^2\theta$

11. $\cos 2\theta=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

12. $\sec^2\theta=\dfrac{2}{1+\cos 2\theta}$

13. $\dfrac{1-\cos 2\theta}{2}=\sin^2\theta$

14. $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=-\sec 2\theta$

15. $\dfrac{(\sin\theta+\cos\theta)^2}{\sin 2\theta}=\csc 2\theta+1$

9. $1+\sin 2\theta = (\sin\theta+\cos\theta)^2$

$= (\sin\theta+\cos\theta)(\sin\theta+\cos\theta)$ → FOIL

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 + 2\sin\theta\cos\theta$

$= 1+\sin 2\theta$ ✓ ☺

10. $\sin 2\theta = 2\cot\theta \cdot \sin^2\theta$

$= 2 \cdot \frac{\cos\theta}{\sin\theta} \cdot \sin^2\theta$

$= 2\cos\theta\sin\theta$

$= \sin 2\theta$ ✓ ☺

11. $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\tan^2\theta}{\sec^2\theta}$

$= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$

$= \cos^2\theta - \frac{\sin^2\theta/\cos^2\theta}{1/\cos^2\theta}$

$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1}$

$= \cos^2\theta - \sin^2\theta$

$= \cos 2\theta$ ✓ ☺

12. $\sec^2\theta = \frac{2}{1+\cos 2\theta}$

$= \frac{2}{1+2\cos^2\theta-1}$

$= \frac{2}{2\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

$= \sec^2\theta$ ✓ ☺

13. $\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$\frac{1-(1-2\sin^2\theta)}{2} =$

$\frac{1-1+2\sin^2\theta}{2} =$

$\frac{2\sin^2\theta}{2}$

☺ ✓ $\sin^2\theta =$

14. $\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta} = -\sec 2\theta$

$\frac{1}{1-\cos^2\theta-\cos^2\theta} =$

$\frac{1}{1-2\cos^2\theta} =$

$\frac{1}{-\cos 2\theta} =$

✓ ☺ $-\sec 2\theta =$

15. $\frac{(\sin\theta+\cos\theta)^2}{\sin 2\theta} = \csc 2\theta + 1$

FOIL → $\frac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)}{\sin 2\theta} =$

$\frac{\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta}{2\sin\theta\cos\theta} =$

$\frac{1+2\sin\theta\cos\theta}{2\sin\theta\cos\theta} =$

$\frac{1}{2\sin\theta\cos\theta} + \frac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta} =$

$\frac{1}{\sin 2\theta} + 1 =$

☺ ✓ $\csc 2\theta + 1 =$