Unit Vectors WS

Name Key

Write each of the following as the sum of unit vectors; also, find the magnitude and the direction of the vector. Round your answers to the nearest hundredth.

1. Initial pt: $(-3,-5)$; Terminal point: $(5,1)$

$\langle 5+3, 1+5\rangle$
$\langle 8,6\rangle$

Vector as sum of unit vectors: $8i+6j$

Magnitude:
$\sqrt{8^2+6^2}$
$\sqrt{64+36}$
$\sqrt{100}$
10

Direction:
$\theta = \tan^{-1}\left(\frac{6}{8}\right)$
$\theta = \tan^{-1}\left(\frac{3}{4}\right)$
$\theta = 36.87°$

2. Initial point: $(-3,11)$; Terminal point: $(9,40)$

$\langle 9+3, 40-11\rangle = \langle 12, 29\rangle$

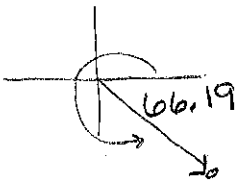
Vector as sum of unit vectors: $12i+29j$

Magnitude:
$\sqrt{12^2+29^2}$
$\sqrt{985}$
31.38

Direction:
$\theta = \tan^{-1}\left(\frac{29}{12}\right)$
$\theta = 67.52°$

3. Initial pt: $(-4.2,5)$; Terminal point: $(3.7,-12.9)$

$\langle 3.7+4.2, -12.9-5\rangle = \langle 7.9, -17.9\rangle$



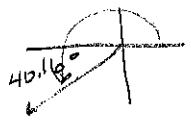
Vector as sum of unit vectors: $7.9i-17.9j$

Magnitude:
$\sqrt{7.9^2+(-17.9)^2}$
$\sqrt{382.82}$
19.57

Direction:
$\theta = \tan^{-1}\left(\frac{-17.9}{7.9}\right)$
$\theta = -66.19°$
$+360$
$\theta = 293.81°$

4. Initial pt: $(1.64,7.21)$; Terminal pt $(-2.33,3.86)$

$\langle -2.33-1.64, 3.86-7.21\rangle = \langle -3.97, -3.35\rangle$



Vector as sum of unit vectors: $-3.97i-3.35j$

Magnitude:
$\sqrt{(-3.97)^2+(-3.35)^2}$
$\sqrt{26.9834}$
5.19

Direction:
$\theta = \tan^{-1}\left(\frac{-3.35}{-3.97}\right)$
$\theta = \tan^{-1}\left(\frac{3.35}{3.97}\right)$
$\theta = 40.16$
$+180$
$220.16°$

Answers:

1) $8\vec{i}+6\vec{j}$; $\|\vec{v}\|=10$; $\theta=36.87°$

2) $12\vec{i}+29\vec{j}$; $\|\vec{v}\|=31.38$; $\theta=67.52°$

3) $7.9\vec{i}-17.9\vec{j}$; $\|\vec{v}\|=19.57$; $\theta=293.81°$

4) $-3.97\vec{i}-3.35\vec{j}$; $\|\vec{v}\|=5.19$; $\theta=220.16°$

For each of the following, find: (a) $-5\vec{u}+2\vec{v}$ (b) $\frac{1}{2}\vec{u}-\vec{v}$ Write answers in the form of the original vectors.

5. $\mathbf{u}=\langle 5,3\rangle$, $\mathbf{v}=\langle -4,0\rangle$

a.) $\langle -25,-15\rangle+\langle 8,0\rangle=\langle -33,-15\rangle$

b.) $\langle \frac{5}{2},\frac{3}{2}\rangle+\langle +4,0\rangle=\langle 6\frac{1}{2},1\frac{1}{2}\rangle$

6. $\mathbf{u}=\mathbf{i}+\mathbf{j}$, $\mathbf{v}=2\mathbf{i}-3\mathbf{j}$

a.) $(-5i-5j)+(4i-6j)=-i-11j$

b.) $(\frac{1}{2}i+\frac{1}{2}j)+(-2i+3j)=-1\frac{1}{2}i+3\frac{1}{2}j$

7. $\mathbf{u}=-9\mathbf{j}$, $\mathbf{v}=-6\mathbf{i}+10\mathbf{j}$

a) $(45j)+(-12i+20j)=-12i+65j$

b) $(-\frac{9}{2}j)+(+6i+10j)=6i-14\frac{1}{2}j$

8. $\mathbf{u}=2\mathbf{i}-\mathbf{j}$, $\mathbf{v}=-\mathbf{i}+\mathbf{j}$

a.) $(-10i+5j)+(-2i+2j)=-12i+7j$

b.) $(i-\frac{1}{2}j)+(+i+j)=2i-1\frac{1}{2}j$

Find a unit vector in the direction of the given vector. Write your answer in the same form as the original vector.

9. $\mathbf{u}=\langle 6,0\rangle$

$\frac{\langle 6,0\rangle}{\sqrt{6^2+0^2}}=\frac{\langle 6,0\rangle}{\sqrt{36}}=\frac{\langle 6,0\rangle}{6}=\langle 1,0\rangle$

10. $\mathbf{v}=\langle -4,4\rangle$

$\frac{\langle -4,4\rangle}{\sqrt{(-4)^2+4^2}}=\frac{\langle -4,4\rangle}{\sqrt{32}}=\frac{\langle -4,4\rangle}{4\sqrt{2}}$

$\langle \frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle=\langle \frac{-\sqrt{2}}{2},\frac{\sqrt{2}}{2}\rangle$

11. $\mathbf{v}=\langle 5,-12\rangle$

$\frac{\langle 5,-12\rangle}{\sqrt{5^2+(-12)^2}}=\frac{\langle 5,-12\rangle}{\sqrt{169}}=\frac{\langle 5,-12\rangle}{13}$

$\langle \frac{5}{13},\frac{-12}{13}\rangle$

12. $\mathbf{v}=4\mathbf{i}-3\mathbf{j}$

$\frac{4i-3j}{\sqrt{4^2+(-3)^2}}=\frac{4i-3j}{\sqrt{25}}=$

$\frac{4i-3j}{5}=\frac{4}{5}i-\frac{3}{5}j$

13. $\mathbf{w}=\mathbf{i}-2\mathbf{j}$

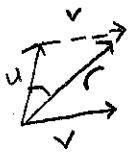
~~(Round to the nearest 100th)~~

$\frac{i-2j}{\sqrt{1^2+(-2)^2}}=\frac{i-2j}{\sqrt{5}}=$

$\frac{1}{\sqrt{5}}i-\frac{2}{\sqrt{5}}j=\frac{\sqrt{5}}{5}i-\frac{2\sqrt{5}}{5}j$

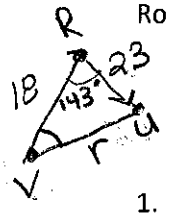
14. $\mathbf{w}=-3\mathbf{i}$

$\frac{-3i}{\sqrt{(-3)^2+0^2}}=\frac{-3i}{\sqrt{9}}=\frac{-3i}{3}=-i$



More Resultant Practice: Find the magnitude of the resultant given the magnitude of $\vec{v}$ and $\vec{u}$ and the measure of the angle θ between the vectors. Also, find the measure of the angle that the resultant makes with u. Round to the nearest hundredth.

$r=\sqrt{u^2+v^2-2uv\cos R}$ $\quad v^2=r^2+u^2-2ru\cos V$



1. $\|\vec{v}\|=23$, $\|\vec{u}\|=18$, $\theta=37^\circ$

$R=180-37=143^\circ$

$r=\sqrt{18^2+23^2-2(18)(23)\cos 143^\circ}$

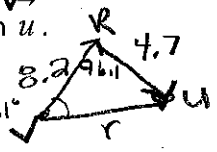
$r=38.91$

$23^2=38.91^2+18^2-2(38.91)(18)\cos V$

$-1308.9881=-1400.76\cos V$

$.93=\cos V$

$V=20.86^\circ$



2. $\|\vec{v}\|=4.7$, $\|\vec{u}\|=8.2$, $\theta=83.9^\circ$

$R=180-83.9=96.1^\circ$

$r=\sqrt{8.2^2+4.7^2-2(8.2)(4.7)\cos 96.1^\circ}$

$r=9.88$

$4.7^2=9.88^2+8.2^2-2(9.88)(8.2)\cos V$

$-142.7644=-162.032\cos V$

$.88=\cos V$

$V=28.23^\circ$

---

5) a) $\langle -33,-15\rangle$ b) $\langle 6.5,1.5\rangle$ 6) a) $-\vec{i}-11\vec{j}$ b) $-1.5\vec{i}+3.5\vec{j}$

7) a) $-12\vec{i}+65\vec{j}$ b) $6\vec{i}-14.5\vec{j}$ 8) a) $-12\vec{i}+7\vec{j}$ b) $2\vec{i}-1.5\vec{j}$

9) $\langle 1,0\rangle$ 10) $\left\langle -\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right\rangle$ 11) $\left\langle \frac{5}{13},-\frac{12}{13}\right\rangle$ 12) $\frac{4}{5}\vec{i}-\frac{3}{5}\vec{j}$ 13) ~~$0.45\vec{i}-0.89\vec{j}$~~ 14) $-\vec{i}$

more resultant practice: 1) $\|\vec{u}+\vec{v}\|=38.91$; $V=20.86^\circ$ 2) $\|\vec{u}+\vec{v}\|=9.88$; $V=28.23^\circ$