Solving Trig Equations WS 1

Name Key

Solve the following equations over $[0, 2\pi)$

1. $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

2. $\sqrt{3}\sec x = 2$

$\sec x = \frac{2}{\sqrt{3}}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

3. $\tan x - \sqrt{3} = 0$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

4. $\sec^2 x - 2 = 0$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5. $2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

6. $2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

7. $\tan x(\tan x - 1) = 0$

$\tan x = 0$ $\quad \tan x - 1 = 0$

$x = 0, \pi$ $\quad \tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

8. $\csc^2 x - 2 = 0$

$\csc^2 x = 2$

$\csc x = \pm\sqrt{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. $\sin x(\sin x + 1) = 0$

$\sin x = 0$ $\quad \sin x + 1 = 0$

$x = 0, \pi$ $\quad \sin x = -1$

$x = \frac{3\pi}{2}$

10. $\sqrt{3}\tan x + 1 = 0$

$\sqrt{3}\tan x = -1$

$\tan x = -\frac{1}{\sqrt{3}}$

$\tan x = -\frac{\sqrt{3}}{3}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

11. $\sec x + 2 = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

12. $\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

13. $\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ $\quad \sec x - 2 = 0$

$\sin x = \text{und.}$ (not possible) $\quad \sec x = 2$

$\cos x = 1/2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

14. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ $\quad \sin x + 1 = 0$

$2\sin x = -1$ $\quad \sin x = -1$

$\sin x = -1/2$ $\quad x = \frac{3\pi}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $2\cos x \sin x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\quad 2\sin x + 1 = 0$

$x = \pi/2, 3\pi/2$ $\quad 2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

16. $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0$ $\quad \sin x - 1 = 0$

$2\sin x = -1$ $\quad \sin x = 1$

$\sin x = -\frac{1}{2}$ $\quad x = \frac{\pi}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

17. $2\cot^2 x \sin x = \cot^2 x$

$2\cot^2 x \sin x - \cot^2 x = 0$

$\cot^2 x(2\sin x - 1) = 0$

$\cot^2 x = 0$ $\quad 2\sin x - 1 = 0$

$\cot x = 0$ $\quad 2\sin x = 1$

$\tan x = \text{und.}$ $\quad \sin x = 1/2$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

18. $\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)(\tan x - 1) = 0$

$\tan x - 1 = 0$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

19. $\sin x \tan x = -\tan x$

$\sin x \tan x + \tan x = 0$

$\tan x(\sin x + 1) = 0$

$\tan x = 0$ $\quad \sin x + 1 = 0$

$\boxed{x = 0, \pi}$ $\quad \sin x = -1$

~~$x = \frac{3\pi}{2}$~~

20. $\sin x - \tan x = 0$

$\sin x = \tan x$

(think: when does sin = tan?)

$\boxed{x = 0, \pi}$

21. $(\cos^2 x - 1)(\cot x + 1) = 0$

$\cos^2 x - 1 = 0$ $\quad \cot x + 1 = 0$

$\cos^2 x = 1$ $\quad \cot x = -1$

$\cos x = \pm 1$ $\quad \boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$

~~$x = 0, \pi$~~

22. $\cos x(\sec x - 2) = 0$

$\cos x = 0$ $\quad \sec x - 2 = 0$

~~$x = \frac{\pi}{2}, \frac{3\pi}{2}$~~ $\quad \sec x = 2$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

23. $\sin^2 x \tan x = \tan x$

$\sin^2 x \tan x - \tan x = 0$

$\tan x(\sin^2 x - 1) = 0$

$\tan x = 0$ $\quad \sin^2 x - 1 = 0$

$x = 0, \pi$ $\quad \sin^2 x = 1$

$\sin x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{x = 0, \pi}$

24. $\cos x = \cot x$

(think: when does cos = cot?)

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

Remember: Undefined