PreCalculus Name Key

Solving Trig Equations with Sum & Difference Identities WS

Solve each of the following over $[0, 2\pi)$.

1. $\sin\left(\frac{\pi}{2} - x\right) = \frac{1}{2}$

$\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x = \frac{1}{2}$

$1(\cos x) - 0(\sin x) = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

2. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

$\left[\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3}\right] + \left[\sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3}\right] = 1$

$\sin x(\frac{1}{2}) + \cos x(\frac{\sqrt{3}}{2}) + \sin x(\frac{1}{2}) - \cos x(\frac{\sqrt{3}}{2}) = 1$

$\frac{1}{2}\sin x + \frac{1}{2}\sin x = 1$

$\sin x = 1$

$x = \frac{\pi}{2}$

3. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$\left[\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6}\right] - \left[\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}\right] = \frac{1}{2}$

$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$

$\cos x = 1/2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

4. $\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$

$\left[\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4}\right] - \left[\cos x\cos\frac{\pi}{4} + \sin x\sin\frac{\pi}{4}\right] = 1$

$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = 1$

$\frac{-2\sqrt{2}}{2}\sin x = 1$

$-\sqrt{2}\sin x = 1$

$\sin x = \frac{1}{-\sqrt{2}}$

$\sin x = -\frac{\sqrt{2}}{2}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

5. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$

$\left[\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6}\right] - \left[\cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6}\right] = 1$

$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = 1$

$-1\sin x = 1$

$\sin x = -1$

$x = \frac{3\pi}{2}$

6. $\cos(x + 3\pi) = \cos x + \sqrt{3}$

$\cos x\cos 3\pi - \sin x\sin 3\pi = \cos x + \sqrt{3}$

$\cos x(-1) - \sin x(0) = \cos x + \sqrt{3}$

$-\cos x = \cos x + \sqrt{3}$

$-2\cos x = \sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

7. $\sin\left(x+\frac{7\pi}{2}\right)=\cos^2 x-2$

$\sin x\cos\frac{7\pi}{2}+\cos x\sin\frac{7\pi}{2}=\cos^2x-2$

$\sin x(0)+\cos x(-1)=\cos^2x-2$

$-\cos x=\cos^2x-2$

$0=\cos^2x+\cos x-2$

$0=(\cos x+2)(\cos x-1)$

$\cos x+2=0$ $\quad\cos x=-2$ ✗

$\cos x-1=0$

$\cos x=1$

$x=0\pi$

8. $\cos\left(x-\frac{\pi}{2}\right)+4\sin x=2\sin^2 x-3$

$[\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}]+4\sin x=2\sin^2x-3$

$[\cos x(0)+\sin x(1)]+4\sin x=2\sin^2x-3$

$\sin x+4\sin x=2\sin^2x-3$

$5\sin x=2\sin^2x-3$

$0=2\sin^2x-5\sin x-3$

$0=(2\sin x+1)(\sin x-3)$

$2\sin x+1=0$ $\quad 2\sin x=-1$ $\quad \sin x=-\frac{1}{2}$

$\sin x-3=0$ $\quad \sin x=3$ ✗

$x=7\pi/6,\ 11\pi/6$

9. $\tan(x+5\pi)=2\tan x+1$

$\left[\frac{\tan x+\tan 5\pi}{1-\tan x\tan 5\pi}\right]=2\tan x+1$

$\left[\frac{\tan x+0}{1-\tan x(0)}\right]=2\tan x+1$

$\left[\frac{\tan x}{1-0}\right]=2\tan x+1$

$\tan x=2\tan x+1$

$-\tan x=1$

$\tan x=-1$

$x=\frac{3\pi}{4},\frac{7\pi}{4}$

10. $\cos\left(x-\frac{3\pi}{2}\right)+\cos^2 x=6+5\sin x$

$[\cos x\cos\frac{3\pi}{2}+\sin x\sin\frac{3\pi}{2}]+\cos^2x=6+5\sin x$

$[\cos x(0)+\sin x(-1)]+\cos^2x=6+5\sin x$

$-\sin x+\cos^2x=6+5\sin x$

$-\sin x+1-\sin^2x=6+5\sin x$

$-5\sin x-6$ $\qquad -6-5\sin x$

$-\sin^2x-6\sin x-5=0$

$\sin^2x+6\sin x+5=0$

$(\sin x+5)(\sin x+1)=0$

$\sin x=-5$ ✗ $\quad \sin x=-1$

$x=\frac{3\pi}{2}$

11. $\sin(\pi+x)=2\sin^2x-\sin x-1$

$\sin\pi\cos x+\cos\pi\sin x=2\sin^2x-\sin x-1$

$0(\cos x)+-1(\sin x)=2\sin^2x-\sin x-1$

$-\sin x=2\sin^2x-\sin x-1$

$0=2\sin^2x-1$

$1=2\sin^2x$

$\frac{1}{2}=\sin^2x$

$\pm\frac{1}{\sqrt{2}}=\sin x$

$\sin x=\pm\frac{\sqrt{2}}{2}$

$x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

12. $\tan(x+\pi)+2\sin(x+\pi)=0$

$\left[\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}\right]+2[\sin x\cos\pi+\cos x\sin\pi]=0$

$\left[\frac{\tan x+0}{1-\tan x(0)}\right]+2[\sin x(-1)+\cos x(0)]=0$

$\tan x+-2\sin x=0$

$\frac{\sin x}{\cos x}-2\sin x=0$

$\sin x\left(\frac{1}{\cos x}-2\right)=0$

$\sin x=0$

$x=0\pi,\pi$

$\frac{1}{\cos x}-2=0$

$\frac{1}{\cos x}=2$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3},\frac{5\pi}{3}$

ANSWERS: 1. $\frac{\pi}{3},\frac{5\pi}{3}$ 2. $\frac{\pi}{2}$ 3. $\frac{\pi}{3},\frac{5\pi}{3}$ 4. $\frac{5\pi}{4},\frac{7\pi}{4}$ 5. $\frac{3\pi}{2}$ 6. $\frac{5\pi}{6},\frac{7\pi}{6}$ 7. 0 8. $\frac{7\pi}{6},\frac{11\pi}{6}$

9. $\frac{3\pi}{4},\frac{7\pi}{4}$ 10. $\frac{3\pi}{2}$ 11. $\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$ 12. $0,\pi,\frac{\pi}{3},\frac{5\pi}{3}$