

Solve over the interval $[0, 2\pi)$.

1. $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0 \quad \cos x - 1 = 0$

$2\cos x = -1 \quad \cos x = 1$

$\cos x = -\frac{1}{2}$

$x = 0\pi$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

2. $\cos 2x + \cos x + 1 = 0$

$2\cos^2 x + \cos x + 1 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x (2\cos x + 1) = 0$

$\cos x = 0 \quad 2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

3. $1 - \cos 2x - \sin x = 0$

$1 - (1 - 2\sin^2 x) - \sin x = 0$

$x - x + 2\sin^2 x - \sin x = 0$

$2\sin^2 x - \sin x = 0$

$\sin x (2\sin x - 1) = 0$

$\sin x = 0 \quad 2\sin x - 1 = 0$

$2\sin x = 1$

$x = 0, \pi$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

4. $\sin^2 x + \cos 2x - \cos x = 0$

$\sin^2 x + \cos^2 x - \sin^2 x - \cos x = 0$

$1 - \sin^2 x - \cos x = 0$

$1 - (1 - \cos^2 x) - \cos x = 0$

$x - x + \cos^2 x - \cos x = 0$

$\cos^2 x - \cos x = 0$

$\cos x (\cos x - 1) = 0$

$\cos x = 0 \quad \cos x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = 1$
 $x = 0\pi$

5. $\sin 2x = \cos x$

$2\sin x \cos x = \cos x$
 $\frac{-\cos x}{-\cos x} \quad \frac{-\cos x}{-\cos x}$

$2\sin x \cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

$\cos x = 0 \quad 2\sin x - 1 = 0$

$2\sin x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

6. $3\cos 2x - 5\cos x = 1$

$3(2\cos^2 x - 1) - 5\cos x = 1$

$6\cos^2 x - 3 - 5\cos x = 1$

$6\cos^2 x - 5\cos x - 4 = 0$

$(3\cos x - 4)(2\cos x + 1) = 0$

$3\cos x - 4 = 0 \quad 2\cos x + 1 = 0$

$2\cos x = -1$

$3\cos x = 4$

$2\cos x = -1$

$\cos x = \frac{4}{3}$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

7. $\sin 2x \sin x + \cos 2x \cos x = 1$

$2 \sin x \cos x \sin x + (1 - 2 \sin^2 x) \cos x = 1$

$2 \sin^2 x \cos x + \cos x - 2 \sin^2 x \cos x = 1$

$\cos x = 1$

$x = 0\pi$

9. $\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x (2 \cos x - 1) = 0$

$\sin x = 0$ $2 \cos x - 1 = 0$

$2 \cos x = 1$

$x = 0\pi, \pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

8. $\cos 2x + 3 \cos x = 1$

$2 \cos^2 x - 1 + 3 \cos x = 1$

$2 \cos^2 x + 3 \cos x - 2 = 0$

$(2 \cos x - 1)(\cos x + 2) = 0$

$2 \cos x - 1 = 0$ $\cos x + 2 = 0$

$2 \cos x = 1$

$\cos x = -2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

10. $\cos 2x + \cos x = 0$

$2 \cos^2 x - 1 + \cos x = 0$

$2 \cos^2 x + \cos x - 1 = 0$

$(2 \cos x - 1)(\cos x + 1) = 0$

$2 \cos x - 1 = 0$ $\cos x + 1 = 0$

$2 \cos x = 1$

$\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \pi$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

11. $\cos \frac{x}{2} - \sin x = 0$

$\sqrt{\frac{1 + \cos x}{2}} - \sin x = 0$

$(\sqrt{\frac{1 + \cos x}{2}})^2 = (\sin x)^2$

$\frac{1 + \cos x}{2} = \sin^2 x$

$1 + \cos x = 2 \sin^2 x$

$1 + \cos x = 2(1 - \cos^2 x)$

$1 + \cos x = 2 - 2 \cos^2 x$

$+2 \cos^2 x - 2 \cos^2 x + \cos x - 1 = 0$

$2 \cos^2 x + \cos x - 1 = 0$

$(2 \cos x - 1)(\cos x + 1) = 0$

$2 \cos x - 1 = 0$ $\cos x + 1 = 0$

$2 \cos x = 1$ $\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \pi$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

12. $\sin \frac{x}{2} + \cos x - 1 = 0$

$\sqrt{\frac{1 - \cos x}{2}} + \cos x - 1 = 0$

$(\sqrt{\frac{1 - \cos x}{2}})^2 = (1 - \cos x)^2$

$\frac{1 - \cos x}{2} = 1 - 2 \cos x + \cos^2 x$

$2 - 4 \cos x + 2 \cos^2 x = 1 - \cos x$

$2 \cos^2 x - 3 \cos x + 1 = 0$

$(2 \cos x - 1)(\cos x - 1) = 0$ $\cos x - 1 = 0$

$2 \cos x - 1 = 0$

$2 \cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x = 1$

$x = 0\pi$

- Answers: 1. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ 2. $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$ 3. $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$ 4. $0, \frac{\pi}{2}, \frac{3\pi}{2}$
 5. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ 6. $\frac{2\pi}{3}, \frac{4\pi}{3}$ 7. 0 8. $\frac{\pi}{3}, \frac{5\pi}{3}$ 9. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$
 10. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 11. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 12. $0, \frac{\pi}{3}, \frac{5\pi}{3}$