Sum & Difference Identities   Name Key

Simplifying, Verifying, Solving Equations WS

Simplify.

1) $\cos 4\theta \cos(-6\theta) - \sin 4\theta \sin(-6\theta)$

$\cos(4\theta + -6\theta)$

$\cos(-2\theta)$

$\boxed{\cos(2\theta)}$

S − C + T −

2) $\sin 6u \cos 5u + \cos 6u \sin 5u$

$\sin(6u + 5u)$

$\boxed{\sin 11u}$

3) $\dfrac{\tan 2v + \tan v}{1 - \tan 2v \tan v}$

$\tan(2v + v)$

$\boxed{\tan 3v}$

4) $\dfrac{\tan 3u - \tan 5u}{1 + \tan 3u \tan 5u}$

$\tan(3u - 5u)$

$\tan(-2u)$

$\boxed{-\tan(2u)}$

S − C + T −

5) $\sin 4\theta \cos 6\theta - \cos 4\theta \sin 6\theta$

$\sin(4\theta - 6\theta)$

$\sin(-2\theta)$

$\boxed{-\sin 2\theta}$

S − C + T −

6) $\cos(-3v)\cos 2v + \sin(-3v)\sin 2v$

$\cos(-3v - 2v)$

$\cos(-5v)$

$\boxed{\cos 5v}$

S − C + T −

Find the exact value of each.

7) $\sin \frac{5\pi}{18} \cos \frac{\pi}{9} - \cos \frac{5\pi}{18} \sin \frac{\pi}{9}$

$\sin\left(\frac{5\pi}{18} - \frac{\pi}{9}\right)$

$\sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$

8) $\cos \frac{11\pi}{9} \cos \frac{17\pi}{36} + \sin \frac{11\pi}{9} \sin \frac{17\pi}{36}$

$\cos\left(\frac{11\pi}{9} - \frac{17\pi}{36}\right) = \cos\left(\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

9) $\dfrac{\tan \frac{17\pi}{9} - \tan \frac{5\pi}{9}}{1 + \tan \frac{17\pi}{9} \tan \frac{5\pi}{9}} = \tan\left(\frac{17\pi}{9} - \frac{5\pi}{9}\right) = \tan\left(\frac{4\pi}{3}\right) = \boxed{\sqrt{3}}$

10) $\dfrac{\tan \frac{\pi}{9} + \tan \frac{5\pi}{36}}{1 - \tan \frac{\pi}{9} \tan \frac{5\pi}{36}} = \tan\left(\frac{\pi}{9} + \frac{5\pi}{36}\right) = \tan \frac{\pi}{4} = \boxed{1}$

11) $\cos \frac{13\pi}{18} \cos \frac{5\pi}{18} - \sin \frac{13\pi}{18} \sin \frac{5\pi}{18}$

$\cos\left(\frac{13\pi}{18} + \frac{5\pi}{18}\right)$

$\cos \pi$

$\boxed{-1}$

12) $\sin \frac{2\pi}{9} \cos \frac{29\pi}{18} + \cos \frac{2\pi}{9} \sin \frac{29\pi}{18}$

$\sin\left(\frac{2\pi}{9} + \frac{29\pi}{18}\right)$

$\sin\left(\frac{11\pi}{6}\right)$

$\boxed{-\frac{1}{2}}$

Verify each identity.

13) $\cos\left(\frac{3\pi}{2} - x\right) = \boxed{-\sin x}$

$\cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$

$0 \cdot \cos x + -1\sin x$

$\boxed{-\sin x}$ ✓

14) $\tan(x + \pi) = \boxed{\tan x}$

$\frac{\tan x + \tan\pi}{1 - \tan x \cdot \tan\pi} = \frac{\tan x + 0}{1 - \tan x \cdot 0} = \frac{\tan x}{1}$

$= \boxed{\tan x}$ ✓

15) $\sin\left(\frac{3\pi}{2} + x\right) = \boxed{-\cos x}$

$\sin\left(\frac{3\pi}{2}\right)\cos x + \cos\left(\frac{3\pi}{2}\right)\sin x$

$-1\cos x + 0 \cdot \sin x$

$\boxed{-\cos x}$ ✓

16) $\sin\left(x - \frac{\pi}{2}\right) = \boxed{-\cos x}$

$\sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2}$

$\sin x \cdot 0 - \cos x \cdot 1$

$\boxed{-\cos x}$ ✓

17) $\tan\left(\frac{\pi}{4} - x\right) = \boxed{\frac{1 - \tan x}{1 + \tan x}}$

$\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \cdot \tan x} = \boxed{\frac{1 - \tan x}{1 + \tan x}}$ ✓

18) $\cos\left(x - \frac{\pi}{2}\right) = \boxed{\sin x}$

$\cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2}$

$\cos x \cdot 0 + \sin x \cdot 1$

$\boxed{\sin x}$ ✓

Solve each equation over one revolution of the unit circle.

19) $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

20) $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$

21) $\tan(x + \pi) + 2\sin(x + \pi) = 0$

22) $2\sin\left(x + \frac{\pi}{2}\right) = \tan\frac{\pi}{3}$

19. $\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}-\left(\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}\right)=\frac{1}{2}$

$$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x-\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)=\frac{1}{2}$$

$$\cancel{\frac{\sqrt{3}}{2}\sin x}+\frac{1}{2}\cos x\cancel{-\frac{\sqrt{3}}{2}\sin x}+\frac{1}{2}\cos x=\frac{1}{2}$$

$$\cos x=\frac{1}{2}$$

$$\boxed{x=\frac{\pi}{3},\frac{5\pi}{3}}$$

20. $\sin(x+\frac{\pi}{2})-\cos(x+\frac{3\pi}{2})=0$

$$\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}-\left(\cos x\cos\frac{3\pi}{2}-\sin x\sin\frac{3\pi}{2}\right)=0$$

$$\cancel{0\cdot\sin x}+1\cdot\cos x-\left(\cancel{0\cdot\cos x}-1\cdot\sin x\right)=0$$

$$\cos x-\sin x=0$$

$$\cos x=\sin x$$

$$\boxed{x=\frac{\pi}{4},\frac{5\pi}{4}}$$

→ just look on unit circle, where does cos(x-value) equal sin (y-value).

21. $\tan(x+\pi)+2\sin(x+\pi)=0$

$$\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}+2\left(\sin x\cos\pi+\cos x\sin\pi\right)=0$$

$$\frac{\tan x+0}{1-\cancel{\tan x\cdot 0}}+2\left(-1\sin x+\cancel{\cos x\cdot 0}\right)=0$$

$$\frac{\tan x}{1}+-2\sin x=0$$

$$\tan x-2\sin x=0$$

$$\frac{\sin x}{\cos x}-2\sin x=0$$

$$\sin x\left(\frac{1}{\cos x}-2\right)=0$$

$\sin x=0$

$\boxed{x=0,\pi}$

$\frac{1}{\cos x}-2=0$

$\sec x-2=0$

$\sec x=2$

$\cos x=\frac{1}{2}$

$\boxed{x=\frac{\pi}{3},\frac{5\pi}{3}}$

22. $2\sin(x+\frac{\pi}{2}) = \tan\frac{\pi}{3}$

$2(\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2}) = \sqrt{3}$

$2(\cancel{\sin x \cdot 0} + \cos x \cdot 1) = \sqrt{3}$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{11\pi}{6}}$