PreCalculus

Simplifying and Verifying with Sum & Difference Identities WS

Name: Key

#1-7. Simplify each of the following. Remember—first step is always to **EXPAND**

1. $\cos\left(\theta-\frac{3\pi}{2}\right)$

$$\cos\theta\cos\frac{3\pi}{2}+\sin\theta\sin\frac{3\pi}{2}$$
$$\cos\theta(0)+\sin\theta(-1)$$
$$0-\sin\theta = \boxed{-\sin\theta}$$

2. $\tan(\theta+\pi)$

$$\frac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi}=\frac{\tan\theta+0}{1-\tan\theta(0)}=\frac{\tan\theta}{1-0}=\frac{\tan\theta}{1}$$
$$=\boxed{\tan\theta}$$

3. $\cos(\pi-\theta)+\sin\left(\frac{\pi}{2}+\theta\right)$

$$\left[\cos\pi\cos\theta+\sin\pi\sin\theta\right]+\left[\sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta\right]$$
$$\left[-1(\cos\theta)+0(\sin\theta)\right]+\left[1(\cos\theta)+0(\sin\theta)\right]$$
$$\left[-\cos\theta+0\right]+\left[\cos\theta+0\right]=-\cos\theta+\cos\theta=\boxed{0}$$

4. $\sin(\theta+\pi)+\cos\left(\theta-\frac{\pi}{2}\right)$

$$\left[\sin\theta\cos\pi+\cos\theta\sin\pi\right]+\left[\cos\theta\cos\frac{\pi}{2}+\sin\theta\sin\frac{\pi}{2}\right]$$
$$\left[\sin\theta(-1)+\cos\theta(0)\right]+\left[\cos\theta(0)+\sin\theta(1)\right]$$
$$\left[-\sin\theta+0\right]+\left[0+\sin\theta\right]$$
$$-\sin\theta+\sin\theta=\boxed{0}$$

5. $\tan(\theta+\pi)-\tan(\pi-\theta)$

$$\left[\frac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi}\right]-\left[\frac{\tan\pi-\tan\theta}{1+\tan\pi\tan\theta}\right]$$
$$\left[\frac{\tan\theta+0}{1-\tan\theta(0)}\right]-\left[\frac{0-\tan\theta}{1+0(\tan\theta)}\right]$$
$$\left[\frac{\tan\theta}{1-0}\right]-\left[\frac{-\tan\theta}{1+0}\right]$$
$$\frac{\tan\theta}{1}-\frac{-\tan\theta}{1}$$
$$\tan\theta++\tan\theta$$
$$\boxed{2\tan\theta}$$

6. $\sin\left(\theta+\frac{\pi}{4}\right)+\sin\left(\theta-\frac{\pi}{4}\right)$

$$\left[\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}\right]+\left[\sin\theta\cos\frac{\pi}{4}-\cos\theta\sin\frac{\pi}{4}\right]$$
$$\sin\theta\left(\frac{\sqrt{2}}{2}\right)+\cancel{\cos\theta\left(\frac{\sqrt{2}}{2}\right)}+\sin\theta\left(\frac{\sqrt{2}}{2}\right)-\cancel{\cos\theta\left(\frac{\sqrt{2}}{2}\right)}$$
$$\frac{\sqrt{2}}{2}\sin\theta+\frac{\sqrt{2}}{2}\sin\theta=\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\sin\theta=\boxed{\sqrt{2}\sin\theta}$$

7. $\cos\left(\theta+\frac{\pi}{4}\right)-\cos\left(\theta-\frac{\pi}{4}\right)$

$$\left[\cos\theta\cos\frac{\pi}{4}-\sin\theta\sin\frac{\pi}{4}\right]-\left[\cos\theta\cos\frac{\pi}{4}+\sin\theta\sin\frac{\pi}{4}\right]$$
$$\left[\cos\theta\left(\frac{\sqrt{2}}{2}\right)-\sin\theta\left(\frac{\sqrt{2}}{2}\right)\right]-\left[\cos\theta\left(\frac{\sqrt{2}}{2}\right)+\sin\theta\left(\frac{\sqrt{2}}{2}\right)\right]$$
$$\cancel{\frac{\sqrt{2}}{2}\cos\theta}-\frac{\sqrt{2}}{2}\sin\theta-\cancel{\frac{\sqrt{2}}{2}\cos\theta}-\frac{\sqrt{2}}{2}\sin\theta$$

$$-\frac{2\sqrt{2}}{2}\sin\theta$$
$$=\boxed{-\sqrt{2}\sin\theta}$$

#8-13. Verify the following identities.

8. $\cos(\pi-\theta)+\sin\left(\frac{\pi}{2}+\theta\right)=0$

$[\cos\pi\cos\theta+\sin\pi\sin\theta]+[\sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta]$

$[-1(\cos\theta)+0(\sin\theta)]+[1(\cos\theta)+0(\sin\theta)]$

$[-\cos\theta+0]+[\cos\theta+0]$

$-\cos\theta+\cos\theta$

0 ✓

9. $\sin(\theta+\pi)+\cos\left(\theta-\frac{\pi}{2}\right)=0$

$[\sin\theta\cos\pi+\cos\theta\sin\pi]+[\cos\theta\cos\frac{\pi}{2}+\sin\theta\sin\frac{\pi}{2}]$

$[\sin\theta(-1)+\cos\theta(0)]+[\cos\theta(0)+\sin\theta(1)]$

$[-\sin\theta+0]+[0+\sin\theta]$

$-\sin\theta+\sin\theta$

0 ✓

10. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$[\sin x\cos y+\cancel{\cos x\sin y}]+[\sin x\cos y-\cancel{\cos x\sin y}]$

$\sin x\cos y+\sin x\cos y$

$2\sin x\cos y$ ✓

11. $\cos(x+y)+\cos(x-y)=2\cos x\cos y$

$[\cos x\cos y-\cancel{\sin x\sin y}]+[\cos x\cos y+\cancel{\sin x\sin y}]$

$\cos x\cos y+\cos x\cos y$

$2\cos x\cos y$ ✓

12. $\tan(x+\pi)-\tan(\pi-x)=2\tan x$

$\left[\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}\right]-\left[\frac{\tan\pi-\tan x}{1+\tan\pi\tan x}\right]$

$\left[\frac{\tan x+0}{1-\tan x(0)}\right]-\left[\frac{0-\tan x}{1+0(\tan x)}\right]$

$\left[\frac{\tan x}{1-0}\right]-\left[\frac{-\tan x}{1+0}\right]$

$\left[\frac{\tan x}{1}\right]-\left[\frac{-\tan x}{1}\right]$

$\tan x++\tan x$

$2\tan x$ ✓

13. $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$

$\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right]$

$\left[\frac{1-\tan\theta}{1+1(\tan\theta)}\right]=\frac{1-\tan\theta}{1+\tan\theta}$ ✓