Review

1. $\tan 450^\circ = \tan(2\cdot 225^\circ) = \frac{2\tan 225^\circ}{1-\tan^2 225^\circ} = \frac{2\cdot 1}{1-(1)^2} = \frac{2}{1-1} = \frac{2}{0} = \boxed{\emptyset}$

2. $\cos\left(\frac{8\pi}{3}\right) = \cos\left(2\cdot\frac{4\pi}{3}\right) = \cos^2\left(\frac{4\pi}{3}\right) - \sin^2\left(\frac{4\pi}{3}\right)$
$= \left(\frac{-1}{2}\right)^2 - \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = \frac{-2}{4} = \boxed{\frac{-1}{2}}$

3. $\csc 600^\circ = \csc(2\cdot 300)$
$\Rightarrow \sin(2\cdot 300) = 2\sin 300^\circ \cos 300^\circ$
$= 2\cdot\frac{-\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$
$\csc(2\cdot 300) = \frac{-2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

4. $\sin 165^\circ = \sin\left(\frac{330^\circ}{2}\right) = \sqrt{\frac{1-\cos 330^\circ}{2}} = \sqrt{\frac{1-(\sqrt{3}/2)}{2}} = \sqrt{\frac{(2/2)-(\sqrt{3}/2)}{2}}$

S	A
T	C

$= \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

5. $\cos\left(\frac{7\pi}{8}\right) = \cos\left(\frac{7\pi/4}{2}\right) = -\sqrt{\frac{1+\cos\frac{7\pi}{4}}{2}} = -\sqrt{\frac{1+\sqrt{2}/2}{2}} = -\sqrt{\frac{2/2+\sqrt{2}/2}{2}}$

S	A
T	C

$= -\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{-\sqrt{2+\sqrt{2}}}{2}}$

6. $\sec\left(\frac{5\pi}{12}\right) = \sec\left(\frac{5\pi/6}{2}\right)$

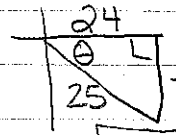
S	A
T	C

$\Rightarrow \cos\left(\frac{5\pi/6}{2}\right) = \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1+(-\sqrt{3}/2)}{2}} = \sqrt{\frac{2/2-\sqrt{3}/2}{2}} =$
$\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$
$= \sec\left(\frac{5\pi/6}{2}\right) = \sqrt{\frac{4}{(2-\sqrt{3})(2+\sqrt{3})}(2+\sqrt{3})} = \sqrt{\frac{8+4\sqrt{3}}{4-3}} = \frac{\sqrt{8+4\sqrt{3}}}{\sqrt{1}} = \boxed{\sqrt{8+4\sqrt{3}}}$

or

or for my big thinkers:
$\sqrt{8+4\sqrt{3}} = \sqrt{4(2+\sqrt{3})} = \boxed{2\sqrt{2+\sqrt{3}}}$

7. $\sin\theta = -\frac{7}{25}$ o h $\quad 270° < \theta < 360°$



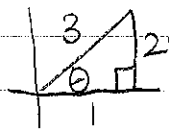
$135° < \frac{\theta}{2} < 180°$

* S | A
T | C

$$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+24/25}{2}} = -\sqrt{\frac{25/25+24/25}{2}} = -\sqrt{\frac{49/25}{2}}$$

$$= -\frac{7/5}{\sqrt{2}} = -\frac{7}{5}\cdot\frac{1}{\sqrt{2}} = -\frac{7}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-7\sqrt{2}}{10}}$$

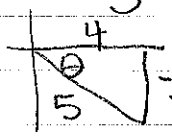
8. $\cos\theta = \frac{1}{3}$ a h $\quad 0° < \theta < 90°$



$1^2 + b^2 = 3^2$
$1 + b^2 = 9$
$b^2 = 8$
$b = 2\sqrt{2}$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2\sqrt{2}}{3}\cdot\frac{1}{3} = \boxed{\frac{4\sqrt{2}}{9}}$$

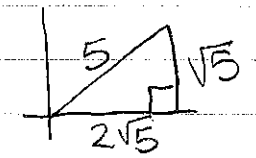
9. $\cos\theta = \frac{4}{5}$ a h $\quad 270° < \theta < 360°$



$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{-3}{5}\cdot\frac{4}{5} = \boxed{\frac{-24}{25}}$$

10. $\cos\theta = \frac{2\sqrt{5}}{5}$ a h $\quad 0° < \theta < 90°$

$(2\sqrt{5})^2 + b^2 = 5^2$
$20 + b^2 = 25$
$b^2 = 5$
$b = \sqrt{5}$



$0° < \frac{\theta}{2} < 45°$

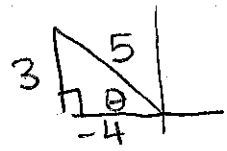
* S | A
T | C

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{2\sqrt{5}}{5}}{2}} = \sqrt{\frac{\frac{5}{5}-\frac{2\sqrt{5}}{5}}{2}} = \sqrt{\frac{\frac{5-2\sqrt{5}}{5}}{2}} =$$

$$= \sqrt{\frac{5-2\sqrt{5}}{5}\cdot\frac{1}{2}} = \sqrt{\frac{5-2\sqrt{5}}{10}} = \frac{\sqrt{5-2\sqrt{5}}}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}$$

$$= \boxed{\frac{\sqrt{50-20\sqrt{5}}}{10}}$$

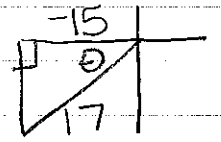
11. $\cos\theta = -\frac{4}{5}$ $\quad 90° < \theta < 180°$



$45° < \frac{\theta}{2} < 90°$ $\quad$ S | A, T | C

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{-4}{5}}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} =$$

$$\sqrt{\frac{9}{5}\cdot\frac{1}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$$

12. 

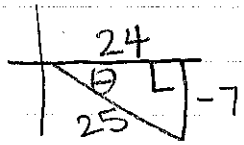
$180° < \theta < 270°$ $\quad \cos\theta = \frac{-15}{17}$ a, h

$90° < \frac{\theta}{2} < 135°$ $\quad$ * S | A, T | C

$$\tan\frac{\theta}{2} = \frac{1-\cos u}{\sin u} = \frac{1-\frac{-15}{17}}{\frac{-8}{17}} = \frac{\frac{17}{17}+\frac{15}{17}}{\frac{-8}{17}} = \frac{\frac{32}{17}}{\frac{-8}{17}} = \frac{32}{17}\cdot\frac{17}{-8} = \frac{-32}{8}$$

$= \boxed{-4}$

13. $\tan x = \frac{-7}{24}$ o, a $\quad \frac{3\pi}{2} < x < 2\pi$

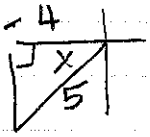


$\frac{3\pi}{4} < \frac{x}{2} < \pi$ $\quad$ * S | A, T | C

$$\cot\left(\frac{x}{2}\right) \rightarrow \tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x} = \frac{1-\frac{24}{25}}{\frac{-7}{25}} = \frac{\frac{25}{25}-\frac{24}{25}}{\frac{-7}{25}} = \frac{\frac{1}{25}}{\frac{-7}{25}}$$

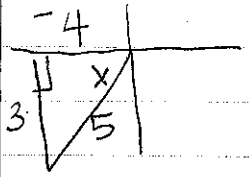
$$= \frac{1}{25}\cdot\frac{25}{-7} = \frac{-1}{7} \quad \text{So, } \cot\left(\frac{x}{2}\right) = \boxed{-7}$$

14. $\cot x = \frac{4}{3}$ a, o $\quad \pi < x < \frac{3\pi}{2}$



$$\sin 2x = 2\sin x\cos x$$

$$= 2\cdot\frac{-3}{5}\cdot\frac{-4}{5} = \boxed{\frac{24}{25}}$$

15. 

$\cot x = \frac{4}{3}$ a $\quad \pi < x < \frac{3\pi}{2}$

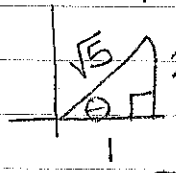
$\cot 2x \Rightarrow \tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot 3/4}{1-(3/4)^2} = \frac{6/4}{16/16 - 9/16} = \frac{3/2}{7/16}$

$= \frac{3}{2}\cdot\frac{16}{7} \div \frac{24}{7}$

$\cot 2x = \boxed{\frac{7}{24}}$

16. $\tan x = \frac{2}{1}$ o a $\quad 0 < x < \frac{\pi}{2}$

$1^2+2^2=c^2$
$1+4=c^2$
$5=c^2$
$\sqrt{5}=c$

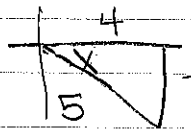


$0 < \frac{x}{2} < \frac{\pi}{4}$ $\quad$ S | A *, T | C

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-1/\sqrt{5}}{2}} = \sqrt{\frac{\sqrt{5}/\sqrt{5}-1/\sqrt{5}}{2}} = \sqrt{\frac{\frac{\sqrt{5}-1}{\sqrt{5}}}{2}}$

$= \sqrt{\frac{\sqrt{5}-1}{\sqrt{5}}\cdot\frac{1}{2}} = \sqrt{\frac{(\sqrt{5}-1)\sqrt{5}}{(2\sqrt{5})\sqrt{5}}} = \sqrt{\frac{5-\sqrt{5}}{10}} = \frac{\sqrt{5-\sqrt{5}}\cdot\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = \boxed{\frac{\sqrt{50-10\sqrt{5}}}{10}}$

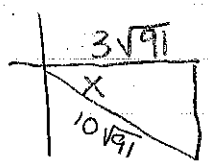
17. $\sin x = \frac{-3}{5}$ o h $\quad \frac{3\pi}{2} < x < 2\pi$



$\frac{3\pi}{4} < x < \pi$ $\quad$ S | A *, T | C

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1-4/5}{-3/5} = \frac{5/5-4/5}{-3/5} = \frac{1/5}{-3/5} = \frac{1}{5}\cdot\frac{-5}{3} = \boxed{\frac{-1}{3}}$

18. $\cot x = \frac{-3\sqrt{91}}{91}$ a $\quad \frac{3\pi}{2} < x < 2\pi$



$(-3\sqrt{91})^2 + (91)^2 = c^2$

$9(91) + 8281 = c^2$

$819 + 8281$

$9100 = c^2$

$10 \cdot 10 \cdot 91$

$10\sqrt{91} = c$

$\frac{3\pi}{4} < \frac{x}{2} < \pi$

x	S	A
	+	C

$\sin\frac{x}{2} =$

$\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\frac{3\sqrt{91}}{10\sqrt{91}}}{2}} = \sqrt{\frac{\frac{10}{10}-\frac{3}{10}}{2}} = \sqrt{\frac{7/10}{2}} = \sqrt{\frac{7}{10}\cdot\frac{1}{2}} = \sqrt{\frac{7}{20}}$

$= \frac{\sqrt{7}}{\sqrt{20}} = \frac{\sqrt{7}}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{35}}{10}}$

19. $\cos 2x + \sin x = -2$

$1 - 2\sin^2 x + \sin x = -2$

$+2 \qquad\qquad +2$

$-2\sin^2 x + \sin x + 3 = 0$

$2\sin^2 x - \sin x - 3 = 0$

$(2\sin x - 3)(\sin x + 1) = 0$

$2\sin x - 3 = 0 \qquad \sin x + 1 = 0$

$2\sin x = 3 \qquad \sin x = -1$

$\sin x = \frac{3}{2}$ $\qquad \boxed{x = \frac{3\pi}{2}}$

20. $\cos 2x - \sin 2x = -2\sin x\cos x$

$2\cos^2 x - 1 - 2\sin x\cos x = -2\sin x\cos x$

$+2\sin x\cos x \qquad +2\sin x\cos x$

$2\cos^2 x - 1 = 0$

$2\cos^2 x = 1$

$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

21. $\cos^2 x - \frac{3}{2}\cos 2x = 0$

$\cos^2 x - \frac{3}{2}(2\cos^2 x - 1) = 0$

$\cos^2 x - 3\cos^2 x + \frac{3}{2} = 0$

$-2\cos^2 x + 3/2 = 0$

$\frac{+1}{2}\cdot -2\cos^2 x = -\frac{3}{2}\cdot -\frac{1}{2}$

$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}} \longrightarrow \cos x = \pm\frac{\sqrt{3}}{2} \longrightarrow \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

22. $2\sin\frac{x}{2} = \sin x$

$2\sqrt{\frac{1-\cos x}{2}}^2 = (\sin x)^2$

$4\left(\frac{1-\cos x}{2}\right) = \sin^2 x$

$2 - 2\cos x = 1 - \cos^2 x$

$+\cos^2 x \quad -1 \qquad -1 \quad +\cos^2 x$

$\cos^2 x - 2\cos x + 1 = 0$

$(\cos x - 1)^2 = 0$

$\cos x - 1 = 0$

$\cos x = 1$

$\boxed{x = 0\pi}$

23. $\sin^2\frac{x}{2} = \cos^2\frac{x}{2}$

$\left(\sqrt{\frac{1-\cos x}{2}}\right)^2 = \left(\sqrt{\frac{1+\cos x}{2}}\right)^2$

$\frac{1-\cos x}{2} = \frac{1+\cos x}{2}$

$2 - 2\cos x = 2 + 2\cos x$

$2 - 2\cos x \quad 2 \quad -2\cos x$

$-4\cos x = 0$

$\cos x = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

24. $\cos 2x - 11\cos x = 5$

$2\cos^2 x - 1 - 11\cos x = 5$

$\quad -5 \qquad\qquad -5$

$2\cos^2 x - 11\cos x - 6 = 0$

$(2\cos x + 1)(\cos x - 6) = 0$

$2\cos x + 1 = 0 \qquad \cos x - 6 = 0$

$2\cos x = -1 \qquad \cos x = 6$

$\cos x = -\frac{1}{2}$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

Remember:

25. $\cos^2\theta - \sin^2\theta = \cos 2\theta$

26. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

27. $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

28. $\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$

25. $\cos^2 x\,\frac{3\pi}{7} - \sin^2\frac{3\pi}{7} = \cos\left(2\cdot\frac{3\pi}{7}\right) = \boxed{\cos\left(\frac{6\pi}{7}\right)}$

26. $\frac{2\tan 31^\circ}{1-\tan^2 31^\circ} = \tan(2\cdot 31^\circ) = \boxed{\tan(62^\circ)}$

27. $\sqrt{\frac{1-\cos\pi/9}{2}} = \sin\left(\frac{\pi/9}{2}\right) = \sin\left(\frac{\pi}{9}\cdot\frac{1}{2}\right) = \boxed{\sin\left(\frac{\pi}{18}\right)}$

28. $\frac{1-\cos 80^\circ}{\sin 80^\circ} = \tan\frac{80^\circ}{2} = \boxed{\tan(40^\circ)}$

29. $\sin 2x = \tan x(1+\cos 2x)$

$= \frac{\sin x}{\cos x}(1 + 2\cos^2 x - 1)$

$= \frac{\sin x}{\cos x}(2\cos^2 x)$

$= \sin x \cdot 2\cos x$

$= 2\sin x \cos x$

$= \sin 2x$ ✓ ☺

30. $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

$= \frac{1-\tan^2 x}{\sec^2 x}$

$= \frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x}$

$= \cos^2 x - \frac{\sin^2 x / \cos^2 x}{1/\cos^2 x}$

$= \cos^2 x - \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$ ✓ ☺