

Final Exam Review 2024 Precalculus:

**Complete all problems on a separate sheet of paper. Answers are provided.
Worked out solutions are within each unit on CTLS.**

Sinusoidal Applications Review (Jan 10)

Name _____

For each application, graph the function, write an equation, and answer the relevant questions using the technology of a graphing calculator.

1) The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The radius of the Ferris wheel is 70 feet. You enter the ride at 10 feet above the ground.

a) Graph and write a function to show how one passenger's height above the ground varies with time as she rides the Ferris wheel.

b) Determine how high above the ground the passenger is 5 minutes into the ride.

c) Determine the number of minutes the passenger has been on the ride when she first reaches a height of 30 feet.

2) Assume that you are aboard a research submarine doing submerged training exercises in the Pacific Ocean. At time $t = 0$, you start porpoising (going alternately deeper and shallower). At time $t = 5$ minutes you are at your deepest depth, -1200 meters. At time $t = 15$ minutes, you next reach your shallowest depth, -300 meters. Assume that the depth varies sinusoidally with time.

a) Graph and write an equation expressing your depth as a function of time.

b) Your submarine can't communicate with ships on the surface when it is deeper than -400 meters. At time $t = 0$, could your submarine communicate? Explain your answer.

c) Between what two nonnegative times is your submarine first able to communicate?

3) As you stop your car at a traffic light, a pebble becomes wedged between your tire treads. When you start moving again, the distance between the pebble and the pavement varies sinusoidally with the distance that you have gone. The period is the circumference of the tire. The diameter of your tire is 26 inches.

a) Graph and write an equation of the function that has **NO phase shift**.

b) What is the pebble's distance from the pavement when you have gone 14 inches?

c) What are the first two distances you have gone when the pebble is 10 inches from the pavement?

4) The height of an object varies sinusoidally over time. A maximum height of 21 inches occurs at 2 hours. The minimum height is 8 inches and occurs at 6 hours.

a) Graph and write an equation to model this situation.

b) What is the height of the object at 22.3 hours?

c) What is the first time after 3 hours that the height of the object is 18 inches?

Answers:

- 1) b) $h(5) = 95.6$ feet c) $h(t) = 30$ feet when $t = 0.9$ minutes
- 2) b) no – you'd be 750 feet below the surface c) between 12.8 minutes and 17.2 minutes
- 3) b) $d(14) = 6.84$ inches c) $x = 17.4$ inches and $x = 64.3$ inches
- 4) b) $h(22.3) = 8.2$ inches c) $t = 3.3$ hours

For each of the following functions, identify the domain and range of the “primary” phase and the period.

| | Domain | Range | Period |
|--------------|--------|-------|--------|
| 1. sine | _____ | _____ | _____ |
| 2. cosine | _____ | _____ | _____ |
| 3. cosecant | _____ | _____ | _____ |
| 4. secant | _____ | _____ | _____ |
| 5. tangent | _____ | _____ | _____ |
| 6. cotangent | _____ | _____ | _____ |

Fill in the blank.

- To graph a secant or cosecant, you would first graph it's _____ function.
(Hint: It's a word that starts with the letter “r”.)
- The reciprocal of secant is _____.
- The reciprocal of cosecant is _____.
- Secant, cosecant, tangent, and cotangent all have undefined values that are represented by a(an) _____ on the graph.

Graph and identify the period, domain, range, and asymptotes.

11. $y = 3 \csc\left(2x - \frac{\pi}{6}\right)$

12. $y = \frac{1}{2} \sec\left(\frac{x}{3} + \frac{\pi}{4}\right)$

13. $y = 3 \tan(2\theta - 40^\circ)$

14. $y = \cot\left(\frac{x}{2} - \pi\right)$

15. $y = 2 \cot(3\theta - 90^\circ) - 2$

16. $y = -\csc\left(\frac{x}{3}\right)$

17. $y = 2 \sec\left(\frac{x}{2} - \frac{\pi}{4}\right)$

18. $y = -3 \tan(4x - \pi)$

Answers:

1. D: $[0, 2\pi]$ R: $[-1, 1]$ pd = 2π

2. D: $[0, 2\pi]$ R: $[-1, 1]$ pd = 2π

3. D: $(0, \pi) \cup (\pi, 2\pi)$ R: $(-\infty, -1] \cup [1, \infty)$ pd = 2π

4. D: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ R: $(-\infty, -1] \cup [1, \infty)$ pd = 2π

5. D: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ R: $(-\infty, \infty)$ pd = π

6. D: $(0, \pi)$ R: $(-\infty, \infty)$ pd = π

7. reciprocal

8. cosine

9. sine

10. asymptote

11. pd = π D: $\left(\frac{\pi}{12}, \frac{7\pi}{12}\right) \cup \left(\frac{7\pi}{12}, \frac{13\pi}{12}\right)$ R: $(-\infty, -3] \cup [3, \infty)$ asy @ $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}$

12. pd = 6π D: $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{15\pi}{4}\right) \cup \left(\frac{15\pi}{4}, \frac{21\pi}{4}\right]$ R: $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ asy @ $\frac{3\pi}{4}, \frac{15\pi}{4}$

13. pd = 90° D: $(-25^\circ, 65^\circ)$ R: $(-\infty, \infty)$ asy @ $-25^\circ, 65^\circ$

14. pd = 2π D: $(2\pi, 4\pi)$ R: $(-\infty, \infty)$ asy @ $2\pi, 4\pi$

Simplify each of the following.

1. $\tan^2 x - \sec^2 x$

2. $\sec^2 x(1 - \cos^2 x)$

3. $\cos x + \tan x \sin x$

4. $\frac{\sin x \cos x}{1 - \sin^2 x}$

5. $\frac{\sin(-x)}{\cos(-x)}$

6. $\cos x \csc x$

7. $\sec^4 x - \tan^4 x$

8. $\frac{\sec^2 x - 1}{\sin^2 x}$

9. $\cot x \sin x$

10. $\sin \beta(\csc \beta - \sin \beta)$

11. $\frac{\cot x}{\csc x}$

12. $\sec \beta \cdot \frac{\sin \beta}{\tan \beta}$

13. $\cot^2 x - \cot^2 x \cos^2 x$

14. $\sin^2 x \sec^2 x - \sin^2 x$

15. $\tan^4 x + 2 \tan^2 x + 1$

16. $\sin^4 x - \cos^4 x$

17. $\sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x$

18. $\tan^2 x - \tan^2 x \sin^2 x$

19. $(\sin x + \cos x)^2$

20. $(\cot x + \csc x)(\cot x - \csc x)$

Answers:

1. -1 2. $\tan^2 x$ 3. $\sec x$ 4. $\tan x$ 5. $-\tan x$ 6. $\cot x$ 7. $\sec^2 x + \tan^2 x$ 8. $\sec^2 x$ 9. $\cos x$ 10. $\cos^2 x$

11. $\cos x$ 12. 1 13. $\cos^2 x$ 14. $\sin^2 x \tan^2 x$ 15. $\sec^4 x$ 16. $\sin^2 x - \cos^2 x$ 17. 1 18. $\sin^2 x$

19. $1 + 2 \sin x \cos x$ 20. -1

Trig Identities (Feb 9)
Verifying Identities WS 2

Name _____

Verify the identity.

1. $\sin^3 \theta + \sin \theta \cos^2 \theta = \sin \theta$

2. $\frac{1 + \sec(-x)}{\sin(-x) + \tan(-x)} = -\csc x$

3. $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

4. $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

5. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

6. $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

7. $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2\csc^2 x$

8. $(\cot^2 \theta + 1)(\sin^2 \theta - 1) = -\cot^2 \theta$

9. $\csc x + \cot x = \frac{\sin x}{1 - \cos x}$

10. $\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

Answers:

You have all of them, right?! (The answers are in the problem!)

PreCalculus (March 13)
Solving Trig Equations Review WS

Name _____

Solve over $[0, 2\pi)$.

1. $\tan x = 2\sin x$

2. $1 + \sin x = 2\cos^2 x$

3. $\sin^2 x = 2\cos x + 2$

4. $\tan x = \cot x$

5. $\csc^2 x = \cot x + 1$

6. $\tan^2 x = -\frac{3}{2}\sec x$

7. $\sin x \tan x = -\tan x$

8. $2\sin^2 x = 3\sin x - 1$

9. $2\sin^2 x = \sqrt{3}\sin x$

10. $\cot^2 x + \csc^2 x = 3$

11. $2\cos x \csc x = \sqrt{3}\csc x$

12. $3\cos x + 3 = 2\sin^2 x$

13. $\tan^2 x = \sqrt{3}\tan x$

14. $(\tan x - 1)(\sec x - 1) = 0$

15. $\sec^2 x - 2\tan x = 0$

16. $(\sin^2 x - 1)(\tan x + 1) = 0$

17. $3\cos x + \sqrt{2} = \cos x$

18. $(\sec^2 x - 2)(\csc x + 1) = 0$

19. $\cot x(\csc x + 2) = 0$

20. $2\cos^2 x - 7\cos x = -3$

21. $6\sin 2x - 3 = 0$

22. $\tan 3x(\tan x - 1) = 0$

23. $3\tan^2 2x = 1$

24. $4\sec 3x + 8 = 0$

Answers:

1. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

2. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

3. π

4. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$

6. $\frac{2\pi}{3}, \frac{4\pi}{3}$

7. $0, \pi$

8. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

9. $0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$

10. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

11. $\frac{\pi}{6}, \frac{11\pi}{6}$

12. $\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

13. $0, \pi, \frac{\pi}{3}, \frac{4\pi}{3}$

14. $0, \frac{\pi}{4}, \frac{5\pi}{4}$

15. $\frac{\pi}{4}, \frac{5\pi}{4}$

16. $\frac{3\pi}{4}, \frac{7\pi}{4}$

17. $\frac{3\pi}{4}, \frac{5\pi}{4}$

18. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

19. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

20. $\frac{\pi}{3}, \frac{5\pi}{3}$

21. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

22. $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

23. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

24. $\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}$

#1-6. Using the sum & difference identities, condense each of the following and express as a trig function of a single angle.

1. $\sin 97^\circ \cos 43^\circ + \cos 97^\circ \sin 43^\circ$

2. $\cos 72^\circ \cos 130^\circ + \sin 72^\circ \sin 130^\circ$

3. $\frac{\tan 140^\circ - \tan 60^\circ}{1 + \tan 140^\circ \tan 60^\circ}$

4. $\sin \frac{\pi}{5} \cos \frac{2\pi}{3} - \cos \frac{\pi}{5} \sin \frac{2\pi}{3}$

5. $\cos \frac{\pi}{6} \cos \frac{\pi}{7} - \sin \frac{\pi}{6} \sin \frac{\pi}{7}$

6. $\frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$

#7-8. Use the sum & difference identities with unit circle values to find exact answers for the following:

7. $\tan(-105^\circ)$

8. $\sin 345^\circ$

#9-11. Given: $\csc \alpha = \frac{13}{5}$, $\frac{\pi}{2} \leq \alpha \leq \pi$, and $\tan \beta = -\frac{3}{4}$, $\frac{3\pi}{2} \leq \beta \leq 2\pi$, find the following:

9. $\sin(\alpha - \beta)$

10. $\cos(\beta + \alpha)$

11. $\tan(\alpha - \beta)$

#12-13. If $\sin \theta = -\frac{3}{5}$ and θ is in the third quadrant, find the following:

12. $\cos(\theta + \frac{\pi}{3})$

13. $\tan 2\theta$

#14-18. Verify the following identities.

14. $\sin(\pi - x) = \sin x$

15. $\sin(\frac{3\pi}{2} + x) = -\cos x$

16. $\cos(30^\circ - x) + \cos(30^\circ + x) = \sqrt{3} \cos x$

17. $\frac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta} = \cot \alpha - \cot \beta$

18. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$

#19-21. Solve each of the following equations over the interval $[0, 2\pi)$.

19. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

20. $\tan(x + \pi) + 2 \sin(x + \pi) = 0$

21. $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$

Answers: 1. $\sin 140^\circ$ 2. $\cos 58^\circ$ 3. $\tan 80^\circ$ 4. $-\sin\left(\frac{7\pi}{15}\right)$

5. $\cos\left(\frac{13\pi}{42}\right)$ 6. $\tan\frac{7\pi}{12}$ 7. $2+\sqrt{3}$ 8. $\frac{\sqrt{2}-\sqrt{6}}{4}$ 9. $-\frac{16}{65}$

10. $-\frac{33}{65}$ 11. $\frac{16}{63}$ 12. $\frac{-4+3\sqrt{3}}{10}$ 13. $\frac{24}{7}$ 19. $\frac{\pi}{3}, \frac{5\pi}{3}$

20. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 21. $\frac{\pi}{4}, \frac{5\pi}{4}$

PreCalculus (Apr 23)
Double and Half Angle Identities
Review WS

Name _____

Use a double angle identity to find the exact value of each expression.

1. $\tan 450^\circ$ 2. $\cos \frac{8\pi}{3}$ 3. $\csc 600^\circ$

Use a half angle identity to find the exact value of each expression.

4. $\sin 165^\circ$ 5. $\cos \frac{7\pi}{8}$ 6. $\sec \frac{5\pi}{12}$

Use a double or half angle identity to find the exact value of each expression.

7. Given $\sin \theta = -\frac{7}{25}$ and $270^\circ < \theta < 360^\circ$, find $\cos \frac{\theta}{2}$.

8. Given $\cos \theta = \frac{1}{3}$ and $0^\circ < \theta < 90^\circ$, find $\sin 2\theta$.

9. Given $\cos \theta = \frac{4}{5}$ and $270^\circ < \theta < 360^\circ$, find $\sin 2\theta$.

10. Given $\cos \theta = \frac{2\sqrt{5}}{5}$ and $0^\circ < \theta < 90^\circ$, find $\sin \frac{\theta}{2}$.

11. Given $\cos \theta = -\frac{4}{5}$ and $90^\circ < \theta < 180^\circ$, find $\sin \frac{\theta}{2}$.

12. Given $\cos \theta = -\frac{15}{17}$ and $180^\circ < \theta < 270^\circ$, find $\tan \frac{\theta}{2}$.

13. Given $\tan x = -\frac{7}{24}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\cot \frac{x}{2}$.

14. Given $\cot x = \frac{4}{3}$ and $\pi < x < \frac{3\pi}{2}$, find $\sin 2x$.

15. Given $\cot x = \frac{4}{3}$ and $\pi < x < \frac{3\pi}{2}$, find $\cot 2x$.

16. Given $\tan x = 2$ and $0 < x < \frac{\pi}{2}$, find $\sin \frac{x}{2}$.

17. Given $\sin x = -\frac{3}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\tan \frac{x}{2}$.

18. Given $\cot x = -\frac{3\sqrt{91}}{91}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\sin \frac{x}{2}$.

Solve over the interval $[0, 2\pi)$.

19. $\cos 2x + \sin x = -2$

20. $\cos 2x - \sin 2x = -2 \sin x \cos x$

21. $\cos^2 x - \frac{3}{2} \cos 2x = 0$

22. $2 \sin \frac{x}{2} = \sin x$

23. $\sin^2 \frac{x}{2} = \cos^2 \frac{x}{2}$

24. $\cos 2x - 11 \cos x = 5$

Write as a single trig function of a single angle.

25. $\cos^2 \frac{3\pi}{7} - \sin^2 \frac{3\pi}{7}$

26. $\frac{2 \tan 31^\circ}{1 - \tan^2 31^\circ}$

27. $\sqrt{\frac{1 - \cos \frac{\pi}{9}}{2}}$

28. $\frac{1 - \cos 80^\circ}{\sin 80^\circ}$

Verify each identity.

29. $\sin 2x = \tan x(1 + \cos 2x)$

30. $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

Answers:

1) undefined 2) $-\frac{1}{2}$ 3) $-\frac{2\sqrt{3}}{3}$ 4) $\frac{\sqrt{2-\sqrt{3}}}{2}$ 5) $\frac{-\sqrt{2+\sqrt{2}}}{2}$ 6) $\sqrt{8+4\sqrt{3}}$ 7) $-\frac{7\sqrt{2}}{10}$ 8) $\frac{4\sqrt{2}}{9}$ 9) $-\frac{24}{25}$

10) $\frac{\sqrt{50-20\sqrt{5}}}{10}$ 11) $\frac{3\sqrt{10}}{10}$ 12) -4 13) -7 14) $\frac{24}{25}$ 15) $\frac{7}{24}$ 16) $\frac{\sqrt{50-10\sqrt{5}}}{10}$ 17) $-\frac{1}{3}$ 18) $\frac{\sqrt{35}}{10}$

19) $\frac{3\pi}{2}$ 20) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 21) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ 22) 0 23) $\frac{\pi}{2}, \frac{3\pi}{2}$ 24) $\frac{2\pi}{3}, \frac{4\pi}{3}$

25) $\cos \frac{6\pi}{7}$ 26) $\tan 62^\circ$ 27) $\sin \frac{\pi}{18}$ 28) $\tan 40^\circ$

PreCalculus Quiz Review (May 2)
More Vector Practice -WS

Name _____

You just finished this worksheet, look it over! 😊