

# Sum & Diff Identities

1



2



3



4



5



6



Q1 (2 points)

Rewrite as a single trig function:

$$\sin 23^\circ \cos 17^\circ + \cos 23^\circ \sin 17^\circ$$



A1

$$\begin{aligned} & \sin 23^\circ \cos 17^\circ + \cos 23^\circ \sin 17^\circ \\ &= \sin(23^\circ + 17^\circ) \\ &= \sin(40^\circ) \end{aligned}$$



Q2 (2 points)

True or False?

$$\sin(\theta - \alpha) = \sin \theta - \sin \alpha$$



A2

$$\sin(\theta - \alpha) \neq \sin \theta - \sin \alpha$$

FALSE!!

$$\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$$

Q3 (3 points)

Find the exact value of:

$$\cos 345^\circ$$

$$\text{HINT : } 345^\circ = 210^\circ + 135^\circ$$



A3

$$\begin{aligned}\cos 345^\circ &= \cos(210^\circ + 135^\circ) \\ &= \cos 210^\circ \cos 135^\circ - \sin 210^\circ \sin 135^\circ \\ &= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

Q4 (3 points)

Rewrite as a single trig function:

$$\cos \frac{2\pi}{5} \cos \frac{\pi}{7} - \sin \frac{2\pi}{5} \sin \frac{\pi}{7}$$



A4

$$\begin{aligned} & \cos \frac{2\pi}{5} \cos \frac{\pi}{7} - \sin \frac{2\pi}{5} \sin \frac{\pi}{7} \\ &= \cos \left( \frac{2\pi}{5} + \frac{\pi}{7} \right) = \cos \left( \frac{14\pi}{35} + \frac{5\pi}{35} \right) \\ &= \cos \left( \frac{19\pi}{35} \right) \end{aligned}$$

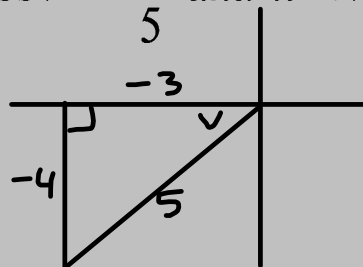
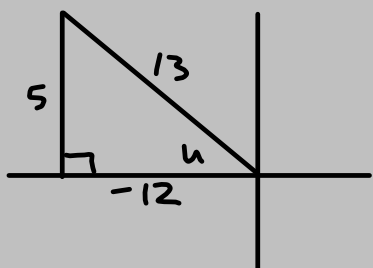
Q5 (4 points)

Given:

$$\sin u = \frac{5}{13} \text{ and } \frac{\pi}{2} < u < \pi \text{ and } \cos v = -\frac{3}{5} \text{ and } \pi < v < \frac{3\pi}{2}$$

find:  $\sin(u - v)$ 

$$\sin u = \frac{5}{13} \text{ and } \frac{\pi}{2} < u < \pi \quad \underline{A5} \quad \cos v = -\frac{3}{5} \text{ and } \pi < v < \frac{3\pi}{2}$$



$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$= \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) - \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right)$$

$$= \left(-\frac{15}{65}\right) - \left(\frac{48}{65}\right) = -\frac{63}{65}$$



## Q6 (6 points)

Given:

$$\sin u = \frac{5}{13} \text{ and } \frac{\pi}{2} < u < \pi \text{ and } \cos v = -\frac{3}{5} \text{ and } \pi < v < \frac{3\pi}{2}$$

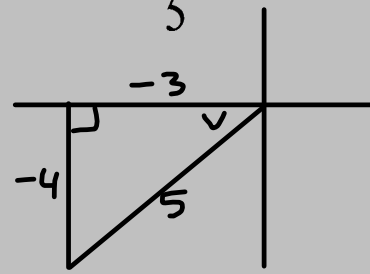
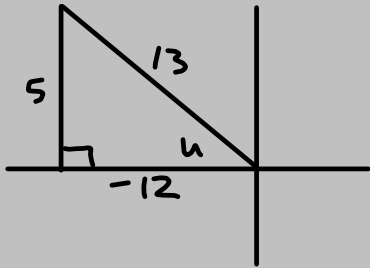
find:  $\tan(u + v)$



$$\sin u = \frac{5}{13} \text{ and } \frac{\pi}{2} < u < \pi$$

A6

$$\cos v = -\frac{3}{5} \text{ and } \pi < v < \frac{3\pi}{2}$$



$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\left(-\frac{5}{12}\right) + \left(\frac{4}{3}\right)}{1 - \left(-\frac{5}{12}\right)\left(\frac{4}{3}\right)}$$

$$= \frac{-\frac{5}{12} + \frac{16}{12}}{\frac{36}{36} + \frac{20}{36}} = \frac{\frac{11}{12}}{\frac{56}{36}} = \frac{11}{12} \cdot \frac{36}{56} = \frac{33}{56}$$

