

Pg. 324 # 1-18, 20-31, 44-49

1.  $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

$$\tan^2\theta \cos^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$$

$$\sin^2\theta \checkmark$$

2.  $\sec^2\theta (1 - \cos^2\theta) = \tan^2\theta$

$$\sec^2\theta - 1$$

$$\tan^2\theta \checkmark$$

3.  $\sin\theta (\sin\theta \cos^2\theta) = \sin^3\theta$

$$\sin\theta (1 - \cos^2\theta)$$

$$\sin\theta (\sin^2\theta)$$

$$\sin^3\theta \checkmark$$

4.  $\csc\theta - \cos\theta \cot\theta = \sin\theta$

$$\frac{1}{\sin\theta} - \frac{\cos\theta \cdot \cos\theta}{\sin\theta}$$

$$\frac{1 - \cos^2\theta}{\sin\theta}$$

$$\frac{\sin^2\theta}{\sin\theta}$$

$$\frac{\sin^2\theta}{\sin\theta}$$

$$\sin\theta$$

$$\sin\theta \checkmark$$

5.  $\cot^2\theta \csc^2\theta - \cot^2\theta = \cot^4\theta$

$$\cot^2\theta (\csc^2\theta - 1)$$

$$\cot^2\theta \cot^2\theta$$

$$\cot^4\theta \checkmark$$

6.  $\tan\theta \csc^2\theta - \tan\theta = \cot\theta$

$$\tan\theta (\csc^2\theta - 1)$$

$$\tan\theta \cdot \cot^2\theta$$

$$\frac{1}{\cot\theta} \cdot \cot^2\theta$$

$$\cot\theta$$

$$\cot\theta \checkmark$$

$$7. \frac{\sec \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \cot \theta$$

↓ mult by  $\cos$ , ↑  $\sin$

$$\frac{\sec \theta \cos \theta - \sin \theta \sin \theta}{\sin \theta \cos \theta}$$

$$\frac{1 - \sin^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\cot \theta$$

✓

$$8. \frac{\sin \theta}{1 - \cos \theta} + \frac{1 - \cos \theta}{\sin \theta} = 2 \csc \theta$$

↑ mult by  $1 + \cos \theta$

$$\frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} + \frac{1 - \cos \theta}{\sin \theta}$$

$$\frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta} + \frac{1 - \cos \theta}{\sin \theta}$$

$$\frac{1 + \cos \theta}{\sin \theta} + \frac{1 - \cos \theta}{\sin \theta}$$

$$\frac{1 + \cos \theta + 1 - \cos \theta}{\sin \theta}$$

$$= \frac{2}{\sin \theta}$$

$$2 \csc \theta$$

✓

$$9. \frac{\cos \theta}{1 + \sin \theta} + \tan \theta = \sec \theta$$

$$\frac{\cos \theta}{1 + \sin \theta}$$

↑  
mult by  $1 - \sin \theta$

$$\frac{\cos \theta (1 - \sin \theta)}{1 - \sin^2 \theta} + \tan \theta$$

$$\frac{\cos \theta (1 - \sin \theta)}{\cos^2 \theta} + \frac{\sin \theta}{\cos \theta}$$

$$\frac{1 - \sin \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

$$\frac{1}{\cos \theta}$$

$\sec \theta \checkmark$

$$10. \frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} = \sin \theta + \cos \theta$$

$$\frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}}$$

$$\frac{\sin \theta}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}}$$

$$\frac{\sin \theta \cdot \sin \theta}{\sin \theta - \cos \theta} + \frac{\cos \theta \cdot \cos \theta}{\cos \theta - \sin \theta}$$

$$\frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\cos \theta - \sin \theta}$$

$$\frac{\sin^2 \theta}{\sin \theta - \cos \theta} - \frac{\cos^2 \theta}{\sin \theta - \cos \theta}$$

$$\frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta}$$

$$\frac{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}{\sin \theta - \cos \theta}$$

$$\sin \theta + \cos \theta \checkmark$$

$$11. \frac{1}{1-\tan^2\theta} + \frac{1}{1-\cot^2\theta} = 1$$

$$\frac{1}{1-\sin^2\theta} + \frac{1}{1-\cos^2\theta}$$

$$\frac{1}{\cos^2\theta - \sin^2\theta} + \frac{1}{\sin^2\theta - \cos^2\theta}$$

$$1 \cdot \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta} + 1 \cdot \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$12. \frac{1}{\csc\theta + 1} + \frac{1}{\csc\theta - 1} = 2\sec^2\theta \sin\theta$$

$$\downarrow \text{mult. by } \csc\theta - 1, \csc\theta + 1$$

$$\frac{\csc\theta - 1}{\csc^2\theta - 1} + \frac{\csc\theta + 1}{\csc^2\theta - 1}$$

$$\frac{2\csc\theta}{\csc^2\theta - 1}$$

$$\frac{2\csc\theta}{\cot^2\theta}$$

$$\frac{2/\sin\theta}{\cos^2\theta/\sin^2\theta}$$

$$\frac{2 \cdot \sin^2\theta}{\sin\theta \cdot \cos^2\theta}$$

$$\frac{2\sin\theta}{\cos^2\theta}$$

$$2\sec^2\theta \sin\theta \checkmark$$

$$13. (\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$$

$$\csc^2\theta - \cot^2\theta$$

$$\cancel{\cot^2\theta} + 1 - \cancel{\cot^2\theta}$$

$$14. \overset{1\checkmark}{\cos^4\theta} - \overset{1\checkmark}{\sin^4\theta} = \overset{1\checkmark}{\cos^2\theta} - \overset{1\checkmark}{\sin^2\theta}$$

$$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$$

$$(\cos^2\theta - \sin^2\theta)(1)$$

$$\cos^2\theta - \sin^2\theta \checkmark$$

$$15. \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$

$\downarrow$  mult by  $1+\sin\theta$ ,  $\downarrow$   $1-\sin\theta$

$$\frac{1+\sin\theta + 1-\sin\theta}{1-\sin^2\theta}$$

$$\frac{2}{\cos^2\theta}$$

$$\boxed{2\sec^2\theta}$$

$$16. \frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$$

$\downarrow$  mult. by  $1-\sin\theta$ ,  $\downarrow$   $1+\sin\theta$

$$\frac{\cos\theta(1-\sin\theta) + \cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$\frac{\cancel{\cos\theta} - \cancel{\cos\theta}\sin\theta + \cancel{\cos\theta} + \cancel{\cos\theta}\sin\theta}{\cos^2\theta}$$

$$\frac{2\cos\theta}{\cos^2\theta}$$

$$\frac{2}{\cos\theta}$$

$$2\sec\theta \checkmark$$

$$17. \csc^4 \theta - \cot^4 \theta = 2 \cot^2 \theta + 1$$

$$(\csc^2 \theta + \cot^2 \theta)(\csc^2 \theta - \cot^2 \theta)$$

$$(1 + \cot^2 \theta + \cot^2 \theta)(1)$$

$$1 + 2 \cot^2 \theta \checkmark$$

$$18. \frac{\csc^2 \theta + 2 \csc \theta - 3}{\csc^2 \theta - 1} = \frac{\csc \theta + 3}{\csc \theta + 1}$$

$$\frac{(\csc \theta + 3)(\csc \theta - 1)}{(\csc \theta + 1)(\csc \theta - 1)}$$

$$\frac{\csc \theta + 3 \checkmark}{\csc \theta + 1}$$

$$20. (\csc \theta + \cot \theta)(1 - \cos \theta) = \sin \theta$$

$$\csc \theta - \cos \theta \csc \theta + \cot \theta - \cos \theta \cot \theta$$

$$\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} - \cos \theta \cdot \frac{\cos \theta}{\sin \theta}$$

$$\frac{1 - \cos^2 \theta}{\sin \theta}$$

$$\frac{1 - \cos^2 \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta}{\sin \theta}$$

$$\sin \theta \checkmark$$

$$21. \sin^2 \theta \tan^2 \theta = \tan^2 \theta - \sin^2 \theta$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$$

↓ mult. by  $\cos^2 \theta$

$$= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta \sin^2 \theta}{\cos^2 \theta}$$

$$= \sin^2 \theta \tan^2 \theta \checkmark$$

$$22. \frac{1 - \tan^2 \theta}{1 - \cot^2 \theta} = \frac{\cos^2 \theta - 1}{\cos^2 \theta}$$

mult  
by  
 $\cos^2 \theta$   
 $\sin^2 \theta$

$$\frac{1 - \sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1 - \cos^2 \theta}{\sin^2 \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}$$

$$\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{-(\cos^2 \theta - \sin^2 \theta)}$$

$$\frac{-\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta - 1}{\cos^2 \theta} \checkmark$$

$$23. \frac{1 + \csc \theta}{\sec \theta} = \cos \theta + \cot \theta$$

$$\frac{1 + \csc \theta}{\sec \theta}$$

$$\cos \theta + \frac{1/\sin \theta}{1/\cos \theta}$$

$$\cos \theta + \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{1}$$

$$\cos \theta + \frac{\cos \theta}{\sin \theta}$$

$$\cos \theta + \cot \theta \checkmark$$

$$\begin{aligned}
 24 \quad (\csc\theta - \cot\theta)^2 &= \frac{1 - \cos\theta}{1 + \cos\theta} \quad \text{mult by } 1 - \cos\theta \\
 &= \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta} \\
 &= \frac{(1 - \cos\theta)^2}{\sin^2\theta} \\
 &= \csc^2\theta (1 - \cos\theta)(1 - \cos\theta) \\
 &= \csc\theta (1 - \cos\theta) (\csc\theta (1 - \cos\theta)) \\
 &= \left( \csc\theta - \frac{\cos\theta}{\sin\theta} \right) \left( \csc\theta - \frac{\cos\theta}{\sin\theta} \right) \\
 &= (\csc\theta - \cot\theta)^2 \quad \checkmark
 \end{aligned}$$

$$25 \quad \frac{1 + \tan^2\theta}{1 - \tan^2\theta} = \frac{1}{2\cos^2\theta - 1}$$

$$\begin{aligned}
 &\frac{\sec^2\theta}{1 - \sin^2\theta} \\
 \xrightarrow{\text{mult by } \cos^2\theta} &\frac{\cos^2\theta}{\cos^2\theta} \cdot \frac{1}{\cos^2\theta - \sin^2\theta} \\
 &\frac{1}{\cos^2\theta - \sin^2\theta} \\
 &\frac{1}{\cos^2\theta - (1 - \cos^2\theta)} \\
 &\frac{1}{\cos^2\theta - 1 + \cos^2\theta} \\
 &\frac{1}{2\cos^2\theta - 1} \quad \checkmark
 \end{aligned}$$

$$26. \tan^2 \theta \cos^2 \theta = 1 - \cos^2 \theta$$

$$\frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$\frac{\sin^2 \theta}{1 - \cos^2 \theta} \checkmark$$

$$27. \sec \theta - \cos \theta = \tan \theta \sin \theta$$

$$\frac{1}{\cos \theta} - \cos \theta \quad \downarrow \text{mult by } \cos \theta$$

$$\frac{1 - \cos^2 \theta}{\cos \theta}$$

$$\frac{\sin^2 \theta}{\cos \theta}$$

$$\frac{\sin \theta \cdot \sin \theta}{\cos \theta}$$

$$\tan \theta \cdot \sin \theta \checkmark$$

$$28. \quad 29. (\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta} \rightarrow 1 - \cos \theta$$

$$= \frac{1 - 2\cos \theta + \cos^2 \theta}{1 - \cos^2 \theta}$$

$$= \frac{1 - 2\cos \theta + \cos^2 \theta}{\sin^2 \theta}$$

$$= \frac{1}{\sin^2 \theta} - \frac{2\cos \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \csc^2 \theta - 2\cot \theta \csc \theta + \cot^2 \theta$$

$$= (\csc \theta - \cot \theta)^2 \checkmark$$

See back (sorry!)

$$\begin{aligned}
 28. \quad 1 - \tan^4 \theta &= 2 \sec^2 \theta - \sec^4 \theta \\
 &= \sec^2 \theta (2 - \sec^2 \theta) \\
 &= (1 + \tan^2 \theta) (2 - (1 + \tan^2 \theta)) \\
 &= (1 + \tan^2 \theta) (1 - \tan^2 \theta) \\
 &= 1 - \tan^4 \theta \checkmark
 \end{aligned}$$

$$\begin{aligned}
 30. \quad 1 + \tan \theta &= \sec \theta \\
 \frac{\sin \theta + \cos \theta}{1 + \sin \theta} & \\
 \text{mult by } \cos \theta & \rightarrow \frac{\sin \theta + \cos \theta}{\cos \theta} \\
 \frac{\sin \theta + \cos \theta}{\cos \theta + \sin \theta} & \\
 \frac{\cos \theta}{\cos \theta} & \\
 \frac{\sin \theta + \cos \theta}{\cos \theta + \sin \theta} & \cdot \frac{1}{1} \\
 \frac{\cos \theta}{\cos \theta} & \cdot \frac{1}{\sin \theta + \cos \theta} \\
 \frac{1}{\cos \theta} & \\
 \sec \theta & \checkmark
 \end{aligned}$$

$$\begin{aligned}
 31. \quad \frac{2 + \csc \theta \sec \theta}{\csc \theta \sec \theta} &= (\sin \theta + \cos \theta)^2 \\
 \frac{2}{\csc \theta \sec \theta} + \frac{\csc \theta \sec \theta}{\csc \theta \sec \theta} & \\
 2 \sin \theta \cos \theta + 1 & \\
 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta & \\
 \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta & \\
 (\sin \theta + \cos \theta)^2 & \checkmark
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \sec^2 \theta + \tan^2 \theta &= \sec^4 \theta - \tan^4 \theta \\
 &= (\sec^2 \theta - \tan^2 \theta) (\sec^2 \theta + \tan^2 \theta) \\
 & \quad | (\sec^2 \theta + \tan^2 \theta) \\
 &= \sec^2 \theta + \tan^2 \theta \checkmark
 \end{aligned}$$



$$\begin{aligned} 48. \sec^4 x &= 1 + 2\tan^2 x + \tan^4 x \\ &= (\tan^2 x + 1)^2 \\ &= (\sec^2 x)^2 \\ &= \sec^4 x \checkmark \end{aligned}$$

$$\begin{aligned} 49. \sec^2 x \csc^2 x &= \sec^2 x + \csc^2 x \\ &= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \\ &\quad \downarrow \text{mult by } \sin^2 x \downarrow \cos^2 x \\ &= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \\ &= \frac{1}{\sin^2 x \cos^2 x} \\ &= \sec^2 x \csc^2 x \checkmark \end{aligned}$$