Pg. 317 # 22-36, 38-47, 51-54

22. $\csc x \sec x - \tan x$

$\frac{1}{\sin x} \cdot \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

↑ mult. by $\sin x$

$\frac{1 - \sin^2 x}{\sin x \cos x}$

$\frac{\cos^2 x}{\sin x \cos x}$

$\frac{\cos x}{\sin x}$

$\boxed{\cot x}$

23. $\csc x - \cos x \cot x$

$\frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x}$

$\frac{1 - \cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x}$

$\boxed{\sin x}$

24. $\sec x \cot x - \sin x$

$\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} - \sin x$

$\frac{1}{\sin x} - \sin x$

↑ mult by $\sin x$

$\frac{1 - \sin^2 x}{\sin x}$

$\frac{\cos^2 x}{\sin x}$

$\frac{\cos x}{\sin x} \cdot \cos x$

$\boxed{\cot x \cos x}$

25. $\frac{\tan x + \sin x \sec x}{\csc x \tan x}$

$\frac{\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}}{\csc x \tan x}$

$\frac{\frac{2\sin x}{\cos x}}{\csc x \tan x}$

$\frac{2\tan x}{\csc x \tan x}$

$\frac{2}{\csc x}$

$2\sin x$

26. $\frac{1-\sin^2 x}{\csc^2 x - 1}$

$\frac{\cos^2 x}{\cot^2 x \cdot \cos^2 x}$

$\frac{\cos^2 x}{\cos^2 x / \sin^2 x}$

$\cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$

$\sin^2 x$

27. $\frac{\csc x \cos x + \cot x}{\sec x \cot x}$

$\frac{\frac{\cos x}{\sin x} + \cot x}{\sec x \cot x}$

$\frac{\cot x + \cot x}{\sec x \cot x}$

$\frac{2\cot x}{\sec x \cot x}$

$\frac{2}{\sec x}$

$2\cos x$

28. $\dfrac{\sec x \csc x - \tan x}{\sec x \csc x}$

$\dfrac{\frac{1}{\cos x}\cdot\frac{1}{\sin x} - \frac{\sin x}{\cos x}}{\sec x \cdot \csc x}$ ← mult by $\sin x$

$\dfrac{1 - \sin^2 x}{\sin x \cos x}$

$\dfrac{\cos^2 x}{\sin x \cancel{\cos x}}$

$\dfrac{\frac{1}{\cos x}\cdot\frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$

$\dfrac{\frac{1}{\cos x \sin x}}{\frac{\cos x}{\sin x}}$

$\dfrac{\cancel{\sin x}}{\cos x}\cdot\dfrac{\cos x \sin x}{1}$

$\boxed{\cos^2 x}$

29. $\dfrac{\sec^2 x}{\cot^2 x + 1}$

$\dfrac{\sec^2 x}{\csc^2 x}$

$\dfrac{1/\cos^2 x}{1/\sin^2 x}$

$\dfrac{1}{\cos^2 x}\cdot\dfrac{\sin^2 x}{1}$

$\dfrac{\sin^2 x}{\cos^2 x}$

$\boxed{\tan^2 x}$

30. $\cot x - \csc^2 x \cot x$

$\cot x(1 - \csc^2 x)$

$\cot x \cdot -\cot^2 x$

$\boxed{-\cot^3 x}$

31. $\cot x - \cos^3 x \csc x$

$\frac{\cos x}{\sin x} - \frac{\cos^3 x}{\sin x}$

$\frac{\cos x - \cos^3 x}{\sin x}$

$\frac{\cos x(1 - \cos^2 x)}{\sin x}$

$\frac{\cos x(\sin^2 x)}{\sin x}$

$\boxed{\cos x \sin x}$

32. $\frac{\cos x}{\sec x + 1} + \frac{\cos x}{\sec x - 1}$

mult by $\sec x - 1$, $\sec x + 1$

$\frac{\cos x(\sec x - 1) + \cos x(\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$

$\frac{1 - \cancel{\cos x} + 1 + \cancel{\cos x}}{\sec^2 x - 1}$

$\frac{2}{\tan^2 x}$

$\boxed{2\cot^2 x}$

33. $\frac{1-\cos x}{\tan x} + \frac{\sin x}{1+\cos x}$

↑ mult by $1-\cos x$

$\frac{1-\cos x}{\tan x} + \frac{\sin x(1-\cos x)}{(1-\cos^2 x)}$

$\frac{1-\cos x}{\tan x} + \frac{\sin x(1-\cos x)}{\sin^2 x}$

$\frac{1-\cos x}{\tan x} + \frac{1-\cos x}{\sin x}$

$\frac{1}{\tan x} - \frac{\cos x}{\tan x} + \frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$\cot x - \frac{\cos x}{\sin x/\cos x} + \frac{1}{\sin x} - \cot x$

$-\cos x \cdot \frac{\cos x}{\sin x} + \frac{1}{\sin x}$

$\frac{-\cos^2 x}{\sin x} + \frac{1}{\sin x}$

$\frac{1-\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x}$

$\boxed{\sin x}$

34. $\frac{1}{\sec x+1} + \frac{1}{\sec x-1}$

↑ mult by $\sec x-1$, ↑ $\sec x+1$

$\frac{\sec x-1}{\sec^2 x-1} + \frac{\sec x+1}{\sec^2 x-1}$

$\frac{2\sec x}{\tan^2 x}$

$\frac{2/\cos x}{\sin^2 x/\cos^2 x}$

$\frac{2}{\cos x} \cdot \frac{\cos^2 x}{\sin^2 x}$

$\frac{2\cos x}{\sin^2 x}$

$\frac{2\cos x}{\sin x} \cdot \frac{1}{\sin x}$

$\boxed{2\cot x \cdot \csc x}$

35. $\dfrac{\cos x \cot x}{\sec x + \tan x} + \dfrac{\sin x}{\sec x - \tan x}$

mult. by $\sec x - \tan x$, $\sec x + \tan x$

$$\frac{\cos x \cot x(\sec x - \tan x) + \sin x(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$$

$$\frac{\cot x - \cos x + \dfrac{\sin x}{\cos x} + \sin x \tan x}{1}$$

$$\boxed{\cot x - \cos x + \tan x + \sin x \tan x}$$

36. $\dfrac{\sin x}{\csc x + 1} + \dfrac{\sin x}{\csc x - 1}$

mult by $\csc x - 1$, $\csc x + 1$

$$\frac{\sin x(\csc x - 1) + \sin x(\csc x + 1)}{\csc^2 x - 1}$$

$$\frac{1 - \sin x + 1 + \sin x}{\cot^2 x}$$

$$\frac{2}{\cot^2 x}$$

$$\boxed{2\tan^2 x}$$

38. $\dfrac{\sin x}{\csc x - \cot x}$

$$\frac{\sin x}{\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}}$$

$$\frac{\sin x}{\dfrac{1 - \cos x}{\sin x}}$$

$$\sin x \cdot \frac{\sin x}{1 - \cos x}$$

$$\frac{\sin^2 x}{1 - \cos x}$$

$$\frac{1 - \cos^2 x}{1 - \cos x}$$

$$\frac{(1 + \cos x)(1 - \cos x)}{1 - \cos x}$$

$$\boxed{1 + \cos x}$$

39. $\frac{\csc x}{1-\sin x}$

↑ mult by $1+\sin x$

$\frac{\csc x(1+\sin x)}{1-\sin^2 x}$

$\frac{\csc x+1}{\cos^2 x}$

$\boxed{\sec^2 x(\csc x+1)}$

40. $\frac{\cot x}{\sec x-\tan x}$

↑ mult by $\sec x+\tan x$

$\frac{\cot x(\sec x+\tan x)}{\sec^2 x-\tan^2 x}$

$\frac{\frac{\cos x}{\sin x}\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)}{1}$

$\frac{1}{\sin x}+1$

$\boxed{\csc x+1}$

41. $\frac{\cot x}{1+\sin x}$

↑ mult by $1-\sin x$

$\frac{\cot x(1-\sin x)}{1-\sin^2 x}$

$\frac{\frac{\cos x}{\sin x}\left(\frac{1-\sin x}{1}\right)}{\cos^2 x}$

$\frac{\frac{\cos x}{\sin x}\left(\frac{1-\sin x}{1}\right)\left(\frac{1}{\cos^2 x}\right)}{}$

$\csc x(1-\sin x)\sec x$

$\boxed{(\csc x-1)\sec x}$

42. $\dfrac{3\tan x}{1-\cos x}$

↑ mult by $1+\cos x$

$$\frac{3\tan x(1+\cos x)}{1-\cos^2 x}$$

$$\frac{3\frac{\sin x}{\cos x}(1+\cos x)}{\sin^2 x}$$

$$\frac{\frac{3\sin x}{\cos x}+3\sin x}{\sin^2 x}$$

$$\frac{3\sin x\left(\frac{1}{\cos x}+1\right)}{\sin^2 x}$$

$$\frac{3\sin x(\sec x+1)}{\sin^2 x}$$

$$3\cancel{\sin x}(\sec x+1)\cdot\frac{1}{\sin^2 x}$$

$$3(\sec x+1)\cdot\frac{1}{\sin x}$$

$$\boxed{3\csc x(\sec x+1)}$$

43 $\frac{2\sin x}{\cot x + \csc x}$

↓ mult by $\cot x - \csc x$

$\frac{2\sin x(\cot x - \csc x)}{\cot^2 x - \csc^2 x}$

$\frac{2\sin x\left(\frac{\cos x}{\sin x} - \frac{1}{\sin x}\right)}{-1}$

$\frac{2\cos x - 2}{-1}$

$-2\cos x + 2$

44. $\frac{\sin x}{1-\sec x}$

↓ mult by $1+\sec x$

$\frac{\sin x(1+\sec x)}{1-\sec^2 x}$

$\frac{\sin x\left(1 + \frac{1}{\cos x}\right)}{-\tan^2 x}$

$\frac{\sin x + \frac{\sin x}{\cos x}}{-\tan^2 x}$

$\frac{\sin x + \tan x}{-\tan^2 x}$

$(\sin x + \tan x) \cdot \frac{-1}{\tan^2 x}$

$\frac{-\sin x}{\tan^2 x} - \frac{1}{\tan x}$

$\frac{-\sin x}{\sin^2 x/\cos^2 x} - \frac{1}{\tan x}$

$-\sin x \cdot \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\tan x}$

$\frac{-\cos^2 x}{\sin x} - \frac{\cos x}{\sin x}$

$\frac{-\cos^2 x - \cos x}{\sin x}$

$-\cos^2 x - \cos x \cdot \frac{1}{\sin x}$

$-\cos x(\cos x + 1) \cdot \frac{1}{\sin x}$

$\frac{-\cos x(\cos x + 1)}{\sin x}$

$-\cot x(\cos x + 1)$

45. $\frac{\cot^2 x \cos x}{\csc x - 1}$

↑ mult by $\csc x + 1$

$\frac{\cot^2 x \cos x (\csc x + 1)}{\csc^2 x - 1}$

$\frac{\cot^2 x (\cos x)(\csc x + 1)}{\cot^2 x}$

$\cos x \left(\frac{1}{\sin x} + 1\right)$

$\frac{\cos x}{\sin x} + \cos x$

$\boxed{\cot x + \cos x}$

46. $\frac{5}{\sec x + 1}$

↑ mult by $\sec x - 1$

$\frac{5(\sec x - 1)}{\sec^2 x - 1}$

$\frac{5(\sec x - 1)}{\tan^2 x}$

$\boxed{5\cot^2 x (\sec x - 1)}$

47. $\frac{\sin x \tan x}{\cos x + 1}$

↑ mult by $\cos x - 1$

$\frac{\sin x \tan x (\cos x - 1)}{\cos^2 x - 1}$

$\frac{\sin x \cdot \frac{\sin x}{\cos x}(\cos x - 1)}{-\sin^2 x}$

$\frac{\sin^2 x \cdot \frac{1}{\cos x}(\cos x - 1)}{-\sin^2 x}$

$\frac{-1}{\cos x}(\cos x - 1)$

$-1 + \frac{1}{\cos x}$

$\boxed{-1 + \sec x}$

51. $\tan x - \csc x \sec x$

$\frac{\sin x}{\cos x} - \frac{1}{\sin x} \cdot \frac{1}{\cos x}$

↓ mult by $\sin x$

$\frac{\sin^2 x - 1}{\sin x \cos}$

$\frac{-\cos^2 x}{\sin x \cos x}$

$\frac{-\cos x}{\sin x}$

$\boxed{-\cot x}$

52. $\cos x + \tan x \sin x$

$\cos x + \frac{\sin x}{\cos x} \cdot \sin x$

↑ mult by $\cos x$

$\frac{\cos^2 x + \sin^2 x}{\cos x}$

$\frac{1}{\cos x}$

$\boxed{\sec x}$

53. $\csc x \tan^2 x - \sec^2 x \csc x$

$\csc x (\tan^2 x - \sec^2 x)$

$\csc x (-1)$

$\boxed{-\csc x}$

54. $\sec x \csc x - \cos x \csc x$

$\csc x (\sec x - \cos x)$

$\csc x \left(\frac{1}{\cos x} - \cos x\right)$ ↓ mult by $\cos x$

$\csc x \left(\frac{1 - \cos^2 x}{\cos x}\right)$

$\frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos x}$ → $\boxed{\tan x}$

$\frac{\sin x}{\cos x}$