21. $\sin^2 x = 3\cos^2 x$

$1 - \cos^2 x = 3\cos^2 x$

$1 = 4\cos^2 x$

$\sqrt{\frac{1}{4}} = \sqrt{\cos^2 x}$

$\cos x = \pm \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

22. $\tan 3x(\tan x - 1) = 0$

$\tan 3x = 0$ $\quad$ $\tan x - 1 = 0$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$ $\quad$ $\tan x = 1$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ $\quad$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$

23. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$3\tan^2 x - 1 = 0$ $\quad$ $\tan^2 x - 3 = 0$

$3\tan^2 x = 1$ $\quad$ $\tan^2 x = 3$

$\tan^2 x = \frac{1}{3}$ $\quad$ $\tan x = \pm\sqrt{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ $\quad$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

24. $\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$ $\quad$ $2\cos x + 1 = 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $\quad$ $2\cos x = -1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\quad$ $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

25. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0$ $\quad$ $\cos^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \pi$

27. $3\tan^3 x = \tan x$

$3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ $\quad$ $3\tan^2 x - 1 = 0$

$x = 0, \pi$ $\quad$ $3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

28. $2\sin^2 x = 2 + \cos x$

$2(1 - \cos^2 x) = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$-2\cos^2 x - \cos x = 0$

$-\cos x(2\cos x + 1) = 0$

$-\cos x = 0$ $\quad$ $2\cos x + 1 = 0$

$\cos x = 0$ $\quad$ $2\cos x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

31. $2\sin x + \csc x = 0$

$2\sin x + \frac{1}{\sin x} = 0$

$2\sin x = -\frac{1}{\sin x}$

$2\sin^2 x = -1$

$\sqrt{\sin^2 x} = \sqrt{\frac{-1}{2}}$

yuck!

$\emptyset$

32. $\sin 2x = -\frac{\sqrt{3}}{2}$

$2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

33. $\csc x + \cot x = 1$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = 1$

$\frac{1+\cos x}{\sin x} = 1$

$1 + \cos x = \sin x$

$(1+\cos x)^2 = (\sin x)^2$

$(1+\cos x)(1+\cos x) = \sin^2 x$

$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$

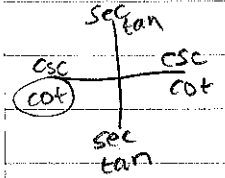
$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$2\cos x = 0 \quad \cos x + 1 = 0$

$\cos x = 0 \quad \cos x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \pi$



34. $\tan 3x = 1$

$3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$

36. $\sec 4x = 2$

$\cos 4x = \frac{1}{2}$

$4x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

37. $\frac{1+\cos x}{1-\cos x} = 0$

$1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

38. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0 \quad \sin x + 1 = 0$

$2\sin x = -1 \quad \sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

39. $2\sec^2 x + \tan^2 x - 3 = 0$

$2(1 + \tan^2 x) + \tan^2 x - 3 = 0$

$2 + 2\tan^2 x + \tan^2 x - 3 = 0$

$3\tan^2 x - 1 = 0$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

40. $\cos x + \sin x \tan x = 2$

$\frac{\cos x + \sin^2 x}{\cos x} = 2$

$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = 2$

$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$

$\frac{1}{\cos x} = 2$

$\cos x = \frac{1}{2}$

$x = \pi/3, 5\pi/3$