PreCalculus

More Sum & Difference Identities WS

Name Key

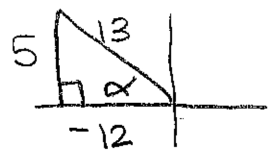
1. Using angles from the unit circle, find the EXACT value of $\cos 255^0$.

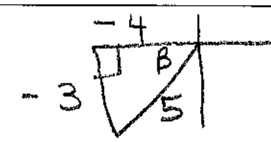
$$\cos(210^\circ+45^\circ)=\cos 210^\circ \cos 45^\circ - \sin 210^\circ \sin 45^\circ$$
$$= \frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{-1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{-\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$$

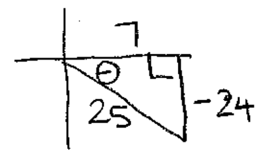
#2-6. Find the following given: α is in quadrant II and $\csc\alpha = \frac{13}{5}$ $\frac{h}{o}$

β is in quadrant III and $\cot\beta = \frac{4}{3}$ $\frac{a}{o}$

θ is in quadrant IV and $\sec\theta = \frac{25}{7}$ $\frac{h}{a}$







$7^2+b^2=25^2$
$49+b^2=625$
$b^2=576$
$b=24$

2. $\cos(\beta-\alpha)$

$\cos\beta\cos\alpha + \sin\beta\sin\alpha$

$\frac{-4}{5}\cdot\frac{-12}{13} + \frac{-3}{5}\cdot\frac{5}{13} = \frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}}$

3. $\sin(\alpha+\beta)$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\frac{5}{13}\cdot\frac{-4}{5} + \frac{-12}{13}\cdot\frac{-3}{5} = \frac{-20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$

4. $\tan(\beta+\theta)$

$\frac{\tan\beta+\tan\theta}{1-\tan\beta\tan\theta} = \frac{\frac{3}{4}+\frac{-24}{7}}{1-\frac{3}{4}\cdot\frac{-24}{7}} = \frac{\frac{21}{28}+\frac{-96}{28}}{1+\frac{72}{28}} = \frac{\frac{-75}{28}}{\frac{28}{28}+\frac{72}{28}} = \frac{\frac{-75}{28}}{\frac{100}{28}} = \frac{-75}{\cancel{28}}\cdot\frac{\cancel{28}}{100} = \frac{-75}{100} = \boxed{\frac{-3}{4}}$

5. $\sin\left(\theta-\frac{7\pi}{6}\right)$

$\sin\theta\cos\frac{7\pi}{6} - \cos\theta\sin\frac{7\pi}{6}$

$\frac{-24}{25}\cdot\frac{-\sqrt{3}}{2} - \frac{7}{25}\cdot\frac{-1}{2}$

$\frac{24\sqrt{3}}{50}+\frac{7}{50}$

$\boxed{\frac{24\sqrt{3}+7}{50}}$

6. $\cos\left(\frac{5\pi}{3}+\alpha\right)$

$\cos\frac{5\pi}{3}\cos\alpha - \sin\frac{5\pi}{3}\sin\alpha$

$\frac{1}{2}\cdot\frac{-12}{13} - \left(-\frac{\sqrt{3}}{2}\cdot\frac{5}{13}\right)$

$\frac{-12}{26}+\frac{5\sqrt{3}}{26}$

$\boxed{\frac{-12+5\sqrt{3}}{26}}$

7. Simplify: $\sin\left(\frac{3\pi}{2}+x\right)$

$\sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$

$-1\cdot\cos x + \cancel{0\cdot\sin x} = \boxed{-\cos x}$

#8-10. Solve each of the following equations over the interval $[0, 2\pi)$

8. $\cos\left(\frac{5\pi}{4} - x\right) = \cos\left(\frac{5\pi}{4} + x\right) - 1$

$\cos\frac{5\pi}{4}\cos x + \sin\frac{5\pi}{4}\sin x = \cos\frac{5\pi}{4}\cos x - \sin\frac{5\pi}{4}\sin x - 1$

$\cancel{-\frac{\sqrt{2}}{2}\cos x} - \frac{\sqrt{2}}{2}\sin x = \cancel{-\frac{\sqrt{2}}{2}\cos x} + \cancel{\frac{\sqrt{2}}{2}\sin x} - 1$

$-\frac{\sqrt{2}}{2}\sin x \qquad\qquad \cancel{-\frac{\sqrt{2}}{2}\sin x}$

$-\sqrt{2}\sin x = -1$

$\sin x = \frac{1}{\sqrt{2}}$

$\sin x = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}}$

9. $2\cos\left(x - \frac{3\pi}{2}\right) = \cot\frac{\pi}{6}$

$2\left[\cos x\cos\frac{3\pi}{2} + \sin x\sin\frac{3\pi}{2}\right] = \sqrt{3}$

$2[\cancel{\cos x \cdot 0} + \sin x \cdot -1] = \sqrt{3}$

$2(-\sin x) = \sqrt{3}$

$-2\sin x = \sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$

10. $\sin\left(x + \frac{7\pi}{2}\right) + 5 - 5\cos x = 8\cos^2 x$

$\sin x\cos\frac{7\pi}{2} + \cos x\sin\frac{7\pi}{2} + 5 - 5\cos x = 8\cos^2 x$

$\cancel{\sin x \cdot 0} + \cos x \cdot -1 + 5 - 5\cos x = 8\cos^2 x$

$-\cos x + 5 - 5\cos x = 8\cos^2 x$

$5 - 6\cos x = 8\cos^2 x$

$0 = 8\cos^2 x + 6\cos x - 5$

$0 = (4\cos x + 5)(2\cos x - 1)$

$4\cos x + 5 = 0 \qquad 2\cos x - 1 = 0$

$4\cos x = -5 \qquad 2\cos x = 1$

$\cos x = \cancel{-\frac{5}{4}} \qquad \cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$