

$$1. \sin 2x = -\sqrt{3} \cos x$$

$$2 \sin x \cos x + \sqrt{3} \cos x = 0$$

$$\cos x (2 \sin x + \sqrt{3}) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

↑  
not between  
 $[\pi, \frac{3\pi}{2}]$

$$2 \sin x + \sqrt{3} = 0$$

$$2 \sin x = -\sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$2. \tan \frac{x}{2} + \cot x = \sqrt{2}$$

$$\frac{1 - \cos x}{\sin x} + \frac{\cos x}{\sin x} = \sqrt{2}$$

$$\frac{1}{\sin x} = \sqrt{2}$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

↑  
not between  $[\pi/2, 2\pi)$

$$3. \tan 2x + \tan x = 0$$

$$\frac{2 \tan x}{1 - \tan^2 x} + \tan x = 0$$

$$\frac{2 \tan x}{1 - \tan^2 x} - \frac{\tan x}{1} = 0$$

$$\frac{2 \tan x}{1 - \tan^2 x} - \frac{\tan x}{1} = 0$$

$$-2 \tan x + \tan^3 x = 2 \tan x$$

$$-2 \tan x \quad -2 \tan x$$

$$-3 \tan x + \tan^3 x = 0$$

$$\tan x (-3 + \tan^2 x) = 0$$

$$\tan x = 0 \quad -3 + \tan^2 x = 0$$

$$x = \pi, 2\pi$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$

not between

$$[\pi/2, \pi]$$

$$x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$$

$$4. \sin(x - \pi/3) = \cos(x - \pi/6) - \frac{\sqrt{6}}{2}$$

$$\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} = \cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6} - \frac{\sqrt{6}}{2}$$

$$\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x = \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x - \frac{\sqrt{6}}{2}$$

$$-\frac{\sqrt{3}}{2} \cos x - \frac{\sqrt{3}}{2} \cos x = -\frac{\sqrt{6}}{2} + \frac{1}{2} \sin x - \frac{1}{2} \sin x$$

$$-\sqrt{3} \cos x = -\frac{\sqrt{6}}{2}$$

$$\frac{1}{\sqrt{3}} \cdot \sqrt{3} \cos x = -\frac{\sqrt{6}}{2} \cdot \frac{1}{\sqrt{3}}$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \pi/4, 7\pi/4 \text{ not between } [0, 3\pi/2]$$

$$5 \quad 4 \cos^2 \frac{x}{2} - 2 \sin^2 x = \cos x + 1$$

$$4 \left( \frac{1 + \cos x}{2} \right)^2 - 2 \sin^2 x = \cos x + 1$$

$$2 \quad 4 \left( \frac{1 + \cos x}{2} \right) - 2 \sin^2 x = \cos x + 1$$

$$2 + 2 \cos x - 2(1 - \cos^2 x) = \cos x + 1$$

$$\begin{array}{r} 2 + 2 \cos x - 2 + 2 \cos^2 x = \cos x + 1 \\ -1 \quad -\cos x \qquad \qquad \qquad -\cos x \quad -1 \end{array}$$

$$2 \cos^2 x + \cos x - 1 = 0$$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$2 \cos x - 1 = 0 \quad \cos x + 1 = 0$$

$$2 \cos x = 1$$

$$\cos x = -1$$

$$\cos x = 1/2$$

$$x = \pi$$

$$x = \pi/3, \boxed{5\pi/3}$$

not between  $[\pi, 2\pi)$

b.  $1 - \sin x = \cos 2x$

$$\begin{array}{r} 1 - \sin x = 1 - 2 \sin^2 x \\ 1 + 2 \sin^2 x - 1 + 2 \sin^2 x \end{array}$$

$$2 \sin^2 x - \sin x = 0$$

$$\sin x (2 \sin x - 1) = 0$$

$$\sin x = 0 \quad 2 \sin x - 1 = 0$$

$$x = 0, \pi, \pi$$

$$2 \sin x = 1$$

$$\sin x = 1/2$$

$$x = \pi/6, \boxed{5\pi/6}$$

not between  $[0, \pi/2]$

$$7. \sin(x - \frac{\pi}{2}) + \sin 2x = 0$$

$$\sin x \cos \frac{\pi}{2} - \cos x \sin \frac{\pi}{2} + 2 \sin x \cos x = 0$$

$$- \cos x + 2 \sin x \cos x = 0$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}$$

~~$\frac{\pi}{2}$~~

not between

$[\frac{\pi}{2}, \pi]$

$$2 \sin x - 1 = 0$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{5\pi}{6}$$

$$8. \tan(x - \frac{\pi}{4}) \cdot \sec^2 x = 0$$

$$\tan x - \tan \frac{\pi}{4} \cdot \sec^2 x = 0$$

$$1 + \tan x \tan \frac{\pi}{4}$$

$$\tan x - 1 \cdot \sec^2 x = 0$$

$$1 + \tan x$$

$$\tan x - 1 = 0$$

$$\sec^2 x = 0$$

$$\tan x = 1$$

$$\sec x = 0$$

$$x = \frac{\pi}{4} \left( \frac{5\pi}{4} \right)$$

$$\cos x = 0$$

not between  $[\pi, 2\pi)$

not possible

$$9. \sin(x+\pi) - 2\cos^2 x = 2\cos(x - \frac{\pi}{2})$$

$$\sin x \cos \pi + \cancel{\cos x \sin \pi} - 2\cos^2 x = 2[\cancel{\cos x \cos \frac{\pi}{2}} + \sin x \sin \frac{\pi}{2}]$$

$$-\sin x - 2\cos^2 x = 2[\sin x]$$

$$-\sin x - 2(1 - \sin^2 x) = 2\sin x$$

$$-\sin x - 2 + 2\sin^2 x = 2\sin x$$

$$\begin{array}{r} -2\sin x \\ \hline -2\sin x \end{array}$$

$$2\sin^2 x - 3\sin x - 2 = 0$$

$$(2\sin x + 1)(\sin x - 2) = 0$$

$$2\sin x + 1 = 0 \quad \sin x - 2 = 0$$

$$2\sin x = -1 \quad \sin x = 2$$

$$\sin x = -\frac{1}{2} \quad \text{not possible}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

not between  $[\frac{3\pi}{2}, 2\pi)$

$$10. \cos 2x + \cos x = 0$$

$$2\cos^2 x - 1 + \cos x = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0$$

$$\cos x + 1 = 0$$

$$2\cos x = 1$$

$$\cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$x = \pi$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

not between  $[\pi, 2\pi)$

$$11. 2\sin^2 \frac{x}{2} - 4\cos^2 x = -\cos x$$

$$2\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 - 4\cos^2 x = -\cos x$$

$$2\left(\frac{1-\cos x}{2}\right) - 4\cos^2 x = -\cos x$$

$$\begin{array}{r} 1 - \cos x - 4\cos^2 x = -\cos x \\ -1 + \cos x \qquad \qquad \qquad + \cos x \quad -1 \end{array}$$

$$-4\cos^2 x = -1$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{4}}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = \cancel{\pi/3}, \boxed{2\pi/3}, \cancel{7\pi/6}, \cancel{11\pi/6}$$

↓ not between  $[\pi/2, \pi]$

$$12. 6\sin^2 x + \cos 2x - 3 = 0$$

$$6(1 - \cos^2 x) + 2\cos^2 x - 1 - 3 = 0$$

$$6 - 6\cos^2 x + 2\cos^2 x - 1 - 3 = 0$$

$$-4\cos^2 x + 2 = 0$$

$$-4\cos^2 x = -2$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$$x = \cancel{\pi/4}, \cancel{3\pi/4}, \boxed{5\pi/4, 7\pi/4}$$

↑  
not between  $[\pi, 2\pi)$

$$13. \sqrt{2} \cos x + \sin 2x = 0$$

$$\sqrt{2} \cos x + 2 \sin x \cos x = 0$$

$$\cos x (\sqrt{2} + 2 \sin x) = 0$$

$$\cos x = 0$$

$$x = \cancel{\frac{\pi}{2}}, \boxed{\frac{3\pi}{2}}$$

not between

$$\left[ \frac{3\pi}{2}, 2\pi \right]$$

$$\sqrt{2} + 2 \sin x = 0$$

$$2 \sin x = -\sqrt{2}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \cancel{\frac{5\pi}{4}}, \boxed{\frac{7\pi}{4}}$$

$$14. \cos x - \cos \frac{x}{2} = 0$$

$$\cos x - \pm \sqrt{\frac{1 + \cos x}{2}} = 0$$

$$L \quad \left( \pm \sqrt{\frac{1 + \cos x}{2}} \right)^2 = (-\cos x)^2$$

$$\frac{1 + \cos x}{2} = \frac{\cos^2 x}{1}$$

$$1 + \cos x = 2 \cos^2 x$$

$$-2 \cos^2 x + \cos x + 1 = 0$$

$$-2 \cos^2 x + \cos x + 1 = 0$$

$$2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$2 \cos x + 1 = 0$$

$$\cos x - 1 = 0$$

$$2 \cos x = -1$$

$$\cos x = 1$$

$$\cos x = -\frac{1}{2}$$

$$x = \cancel{0}$$

$$x = \cancel{\frac{2\pi}{3}}, \boxed{\frac{4\pi}{3}}$$

not between

$$\left[ \pi, \frac{3\pi}{2} \right]$$