

1.  $\frac{\csc x}{\cot x + \tan x} = \frac{\csc x}{\frac{1}{\tan x} + \frac{\tan x \cdot \tan x}{1 \cdot \tan x}} = \frac{\csc x}{\frac{1}{\tan x} + \frac{\tan^2 x}{\tan x}}$   
 $\frac{\csc x}{\frac{1 + \tan^2 x}{\tan x}} = \frac{\csc x}{\frac{\sec^2 x}{\tan x}} = \frac{\csc x}{\frac{1/\cos^2 x}{\sin x/\cos x}} = \frac{\csc x}{\frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x}}$   
 $\frac{\frac{1}{\sin x} \cdot \cancel{\cos x}}{\frac{1}{\cos x \sin x} \cdot \cancel{\cos x}} = \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\cos x} \sin x}{1} = \boxed{\cos x}$

2.  $\frac{\csc x (1 - \sin^2 x)}{\cos x} = \frac{\csc x \cdot \cos^2 x}{\cos x} = \csc x \cos x = \frac{\cos x}{\sin x} = \boxed{\cot x}$

3.  $\frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = \frac{(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)}{\sec^2 x + \tan^2 x} = \sec^2 x - \tan^2 x = \boxed{1}$

4.  $\cos\left(\frac{\pi}{2} - x\right) \csc(-x) = \sin x (-\csc x) = \boxed{-1}$

5.  $\sec x \cos x - \cos^2 x = 1 - \cos^2 x = \boxed{\sin^2 x}$

6.  $\sin x (\sec x + \csc x) = \sin x \left(\frac{1}{\cos x} + \csc x\right) = \frac{\sin x}{\cos x} + 1$

$\boxed{\tan x + 1}$