

PreCalculus
Half Angle Trig Identities WS

Name Key

Use the figure to find the exact value of each trig function.

S/A*
T/C

$$1. \cos \frac{\theta}{2} = \sqrt{\frac{1+\cos \theta}{2}}$$

$$= \sqrt{\frac{1+12/13}{2}} = \sqrt{\frac{13/13+12/13}{2}} = \sqrt{\frac{25/13}{2}}$$

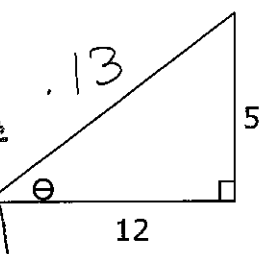
$$= \sqrt{\frac{25}{13} \cdot \frac{1}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

$$2. \sin \frac{\theta}{2} = \sqrt{\frac{1-\cos \theta}{2}}$$

$$= \sqrt{\frac{1-12/13}{2}} = \sqrt{\frac{13/13-12/13}{2}}$$

$$= \sqrt{\frac{1/13}{2}} = \sqrt{\frac{1}{13} \cdot \frac{1}{2}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$$

$$3. \tan \frac{\theta}{2} = \frac{1-\cos \theta}{\sin \theta} = \frac{1-12/13}{5/13} = \frac{13/13-12/13}{5/13} = \frac{1/13}{5/13} = \frac{1}{5}$$



$$4. \sec \frac{\theta}{2} = \frac{1}{\cos \frac{\theta}{2}} = \frac{26}{5}$$

$$5. \csc \frac{\theta}{2} = \frac{1}{\sin \frac{\theta}{2}} = \frac{26}{\sqrt{26}} = \sqrt{26}$$

$$6. \cot \frac{\theta}{2} = \frac{1}{\tan \frac{\theta}{2}} = 5$$

$$7. 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$8. 2 \cos \frac{\theta}{2} \tan \frac{\theta}{2} = 2 \cdot \frac{5\sqrt{26}}{26} \cdot \frac{1}{5} = \frac{10\sqrt{26}}{130} = \frac{\sqrt{26}}{13}$$

$$2 \cdot \frac{\sqrt{26}}{26} \cdot \frac{5\sqrt{26}}{26} = \frac{10(26)}{676} = \frac{260}{676} = \frac{5}{13}$$

Use the half-angle identities to determine the exact values of each function.

$$112.5 \times 2 = 225 \quad \frac{S}{A} \frac{*}{C}$$

$$\frac{\pi}{12} \cdot 2 = \frac{\pi}{6} \quad \frac{S}{A} \frac{*}{C}$$

$$\frac{3\pi}{8} \cdot 2 = \frac{3\pi}{4} \quad \frac{S}{A} \frac{*}{C}$$

$$9. \sin 112.5^\circ = \sin \left(\frac{225^\circ}{2} \right)$$

$$= \sqrt{\frac{1-\cos 225^\circ}{2}} = \sqrt{\frac{1+1/\sqrt{2}}{2}} = \sqrt{\frac{2+1/\sqrt{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$10. \cos \frac{\pi}{12} = \cos \left(\frac{\pi/6}{2} \right)$$

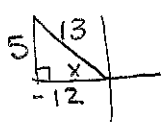
$$= \sqrt{\frac{1+\cos \pi/6}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$11. \tan \frac{3\pi}{8} = \tan \left(\frac{3\pi/4}{2} \right)$$

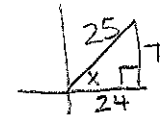
$$= \frac{1-\cos 3\pi/4}{\sin 3\pi/4} = \frac{1+1/\sqrt{2}}{\sqrt{2}/2} = \frac{2+1/\sqrt{2}}{\sqrt{2}/2} = \frac{2\sqrt{2}+1}{1} = 2\sqrt{2}+1$$

Find the exact values of each trig function using the half-angle identities.



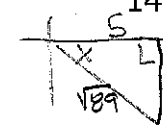
$$12. \text{ Given } \dots \sin x = \frac{5}{13}, \frac{\pi}{4} < x < \frac{\pi}{2} \dots \text{ find } \sin \frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+12/13}{2}}$$

$$= \sqrt{\frac{13/13+12/13}{2}} = \sqrt{\frac{25/13}{2}} = \sqrt{\frac{25}{13} \cdot \frac{1}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$



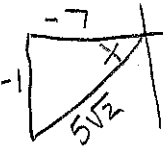
$$13. \text{ Given } \dots \cos x = \frac{7}{25}, 0 < x < \frac{\pi}{2} \dots \text{ find } \cos \frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{25/25+7/25}{2}}$$

$$= \sqrt{\frac{32/25}{2}} = \sqrt{\frac{32}{25} \cdot \frac{1}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$



$$14. \text{ Given } \dots \tan x = -\frac{8}{5}, \frac{3\pi}{4} < x < \pi \dots \text{ find } \tan \frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1-\frac{5}{\sqrt{89}}}{-8/\sqrt{89}} = \frac{\sqrt{89}-5}{-8}$$

$$= \frac{\sqrt{89}-5}{-8} = \frac{\sqrt{89}-5}{-8} = \frac{-\sqrt{89}+5}{8}$$



$$15. \text{ Given } \dots \cot x = \frac{7}{10}, \frac{\pi}{2} < x < \frac{3\pi}{4} \dots \text{ find } \cos \frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+7/5\sqrt{2}}{2}} = -\sqrt{\frac{1-7\sqrt{2}}{10}}$$

$$= -\sqrt{\frac{10-7\sqrt{2}}{10}} = -\sqrt{\frac{10-7\sqrt{2}}{10} \cdot \frac{1}{2}} = -\sqrt{\frac{10-7\sqrt{2}}{20}} = -\frac{\sqrt{10-7\sqrt{2}}}{2\sqrt{5}}$$