

Half-Angle Identities

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u} = \pm \sqrt{\frac{1 - \cos u}{1 + \cos u}}$$

Your answer will NOT be both \pm .
The signs of $\sin(u/2)$ and $\cos(u/2)$ depend on the quadrant in which angle $(u/2)$ lies.

Ex. 1: Use the half-angle formula to find the exact values of ...

a) $\sin 105^\circ$

b) $\cos 105^\circ$

c) $\tan 105^\circ$

Ex. 2: Use the half-angle formula to find the exact values of ...

a) $\sin \frac{\pi}{8}$

b) $\cos \frac{\pi}{8}$

c) $\tan \frac{\pi}{8}$

Ex. 3: Find the exact values of $\sin(u/2)$, $\cos(u/2)$ and $\tan(u/2)$ using the half-angle formulas.

$$\cos u = \frac{5}{13}, \quad \frac{3\pi}{2} < u < 2\pi$$