



Translating Periodic Function

Amplitude
The amplitude, a , vertically stretches or shrinks the graph.

When $0 < a < 1$, the function is shrunk vertically

When $a < 0$, the function is reflected across the x -axis

Period
 $\frac{2\pi}{|b|}$ is the period of the curve. The period is the interval required to complete one full cycle.

4 times as many oscillations in what is normally 1 period (2π). So $\frac{2\pi}{4} = \frac{\pi}{2}$ is the period.

Phase Shift
The horizontal shift, c , translates the graph horizontally. Recall $(x + c)$ moves left.

Sine centered about line $y = 1$ shifted $\frac{\pi}{2}$ to the right

Vertical Shift
The graph oscillates or is centered about the line $y = d$.

Sine stretched 3 above and 3 below the center line $y = 0$. Two waves in 2π so period is $\frac{2\pi}{2} = \pi$ radians.

Try it!

Sine and Cosine Waves

NAME: _____ DATE: _____

Period length of 1 cycle
 $PB = 2\pi$ or $P = \frac{2\pi}{B}$

$y = A \sin(B(x - C) + D)$
 $y = A \cos(B(x - C) + D)$

$|A| = \text{Amplitude}$
 $B = \text{Cycle}$
 $C = \text{Phase Shift}$
 $D = \text{Vertical Shift}$

$y = A \sin(B(x - C) + D)$
MAX = $D + A$
MIN = $D - A$

$y = A \cos(B(x - C) + D)$
COSINE MAX \rightarrow MIDLINE \rightarrow MIN \rightarrow MIDLINE \rightarrow MAX
SINE MIDLINE \rightarrow MAX \rightarrow MIDLINE \rightarrow MIN \rightarrow MIDLINE

PreCalculus - Graphs of the Other Trig Functions

one phase (bx - c = ?)

function	period	start	stop	graph
$y = \csc(x)$	2π	0π	2π	
$y = \sec(x)$	2π	0π	2π	
$y = \tan(x)$	π	$-\pi/2$	$\pi/2$	
$y = \cot(x)$	π	0π	π	

PreCalculus - Trig Identities

Reciprocal:	Pythagorean:	Cofunctions:
$\sin x = \frac{1}{\csc x}$	$\sin^2 x + \cos^2 x = 1$	$\sin \theta = \cos(90^\circ - \theta)$
$\cos x = \frac{1}{\sec x}$	$1 + \tan^2 x = \sec^2 x$	$\cos \theta = \sin(90^\circ - \theta)$
$\tan x = \frac{1}{\cot x}$	$1 + \cot^2 x = \csc^2 x$	$\csc \theta = \sec(90^\circ - \theta)$
		$\sec \theta = \csc(90^\circ - \theta)$
		$\tan \theta = \cot(90^\circ - \theta)$
		$\cot \theta = \tan(90^\circ - \theta)$
Quotient:	Even/Odd:	
$\tan x = \frac{\sin x}{\cos x}$	$\sin(-x) = -\sin x$	$\tan(-x) = -\tan x$
$\cot x = \frac{\cos x}{\sin x}$	$\cos(-x) = \cos x$	$\csc(-x) = -\csc x$
	$\sec(-x) = \sec x$	$\cot(-x) = -\cot x$

Sum/Difference Identities

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Double-Angle Identities

$$\begin{aligned} \cos(2A) &= \cos^2 A - \sin^2 A \\ \cos(2A) &= 1 - 2\sin^2 A \\ \cos(2A) &= 2\cos^2 A - 1 \\ \sin(2A) &= 2\sin A \cos A \\ \tan(2A) &= \frac{2 \tan A}{1 - \tan^2 A} \end{aligned}$$

Half-Angle Identities

$$\begin{aligned} \cos\left(\frac{A}{2}\right) &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \sin\left(\frac{A}{2}\right) &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \tan\left(\frac{A}{2}\right) &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \\ \tan\left(\frac{A}{2}\right) &= \frac{\sin A}{1 + \cos A} \\ \tan\left(\frac{A}{2}\right) &= \frac{1 - \cos A}{\sin A} \end{aligned}$$

Vectors Cheat Sheet

Vector Magnitude

$$\|(a, b)\| = \sqrt{a^2 + b^2}$$

Unit Vector (Normalizing a Vector)

Divide by the magnitude: $\frac{\mathbf{u}}{\|\mathbf{u}\|}$

Adding Vectors

$$(a, b) + (c, d) = (a + c, b + d)$$

$$(at + bt) + (ct + dt) = (a + c)t + (b + d)t$$

Scalar Multiplication

$$a(b, c) = (ab, ac)$$

$$a(bt + ct) = abt + act$$

Vector Angles

Finding the direction of a single vector: $\tan \theta = \frac{y}{x}$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$