

## Discovering Double Angle Trig Identities

### Part 1 – Sine

- a. Find  $\sin 2\theta$  by using the sum identity for  $\sin(\theta + \theta)$  and simplifying.

$$\sin\theta\cos\theta + \sin\theta\cos\theta = 2\sin\theta\cos\theta$$

- b. You have just discovered the “Double Angle Identity for Sine”. ☺ Write the identity in the box below.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

### Part 2 – Tangent

- a. Find  $\tan 2\theta$  by using the sum identity for  $\tan(\theta + \theta)$  and simplifying.

$$\frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta} = \frac{2\tan\theta}{1 - \tan^2\theta}$$

- b. You have just discovered the “Double Angle Identity for Tangent”. ☺ Write the identity in the box below.

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

### Part 3 – Cosine

- a. Find  $\cos 2\theta$  by using the sum identity for  $\cos(\theta + \theta)$  and simplifying.

$$\cos\theta\cos\theta - \sin\theta\sin\theta = \cos^2\theta - \sin^2\theta$$

Remember:  
 $\sin^2\theta + \cos^2\theta = 1$   
 $\sin^2\theta = 1 - \cos^2\theta$   
 $\cos^2\theta = 1 - \sin^2\theta$

- b. You have just discovered the **ONE** “Double Angle Identity for Cosine”. There are **TWO** more!

- c. To find the second “Double Angle Identity for Cosine”, write the first identity below. Use a Pythagorean substitution to replace  $\sin^2\theta$  in your first identity. Simplify. This is a second “Double Angle Identity for Cosine”.

$$\begin{aligned} \cos^2\theta - \sin^2\theta \\ \cos^2\theta - (1 - \cos^2\theta) \\ \cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1 \end{aligned}$$

- d. To find the third “Double Angle Identity for Cosine”, write the first identity below. Use a Pythagorean substitution to replace  $\cos^2\theta$  in your first identity. Simplify. This is a third “Double Angle Identity for Cosine”.

Write all three identities in the box below.

$$\begin{aligned} \cos^2\theta - \sin^2\theta \\ (1 - \sin^2\theta) - \sin^2\theta \\ 1 - 2\sin^2\theta \end{aligned}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$